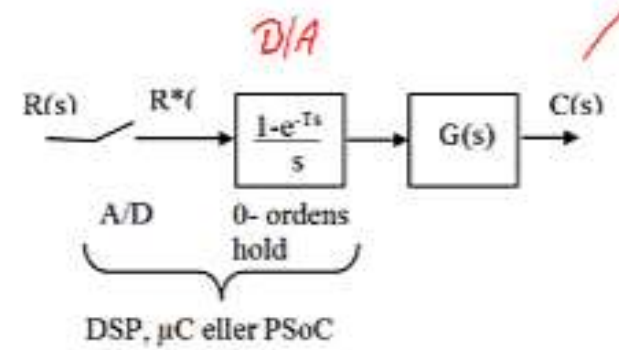


Model af sample / hold og udtryk for åben-sløjfe overføringsfunktionen:



*ikke muligt $\Rightarrow c(t)$ heller ikke muligt
 / tilfredse med kun værdi til sampletidspunkter*

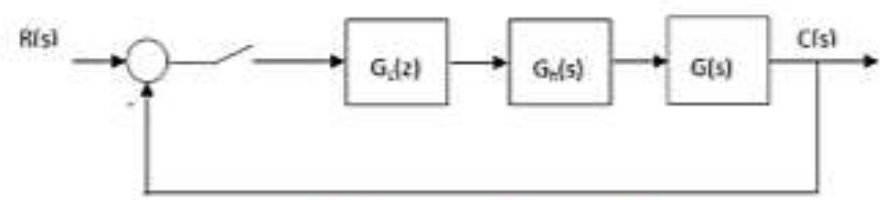
$$C(z) = Z\left\{\frac{1-e^{-Ts}}{s} \cdot G(s)\right\} \cdot R(z) = (1-z^{-1}) \cdot Z\left\{\frac{G(s)}{s}\right\} \cdot R(z)$$

$z = e^{sT}$

$c(nT)$

skal transformeres sammen

Udtryk for lukket-sløjfe overføringsfunktionen:



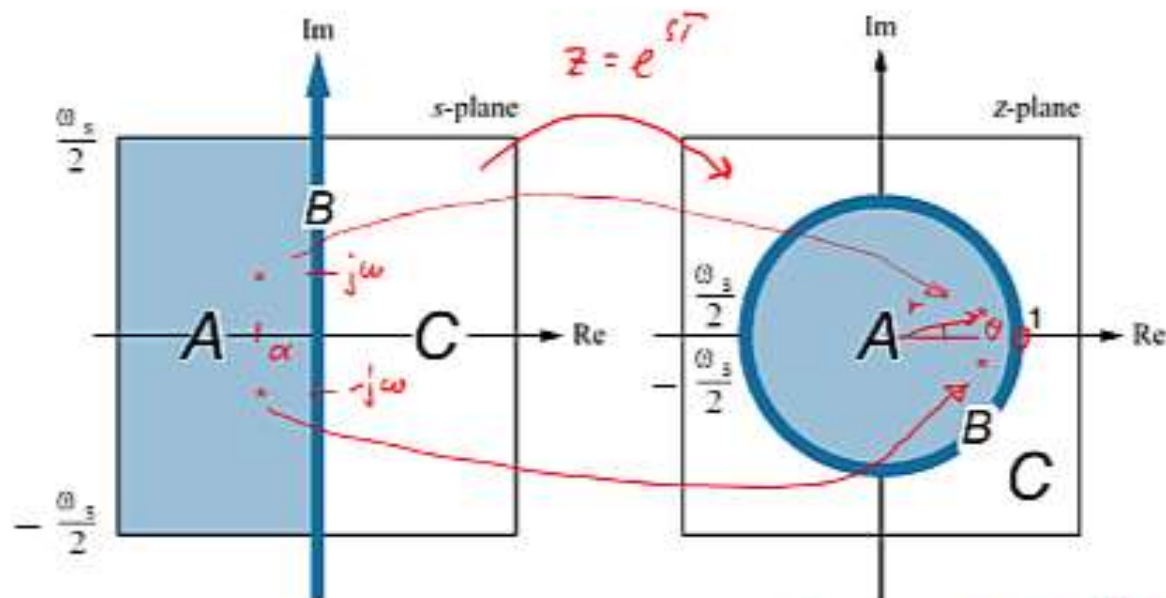
$$\frac{C(z)}{R(z)} = \frac{G_c(z) \cdot \overline{G_h G(z)}}{1 + G_c(z) \cdot \overline{G_h G(z)}}$$

*åben sløjfe
 med kompensering*

*Bodeplot for et $G(z)$?
 $z = e^{sT} \rightarrow e^{j\omega T}$*

på ω_p DC-forst. det samme

Mapping regions of the s-plane onto the z-plane



$$z = e^{sT}, s = \alpha \pm j\omega \Rightarrow z = e^{\alpha T \pm j\omega T} = e^{\alpha T} e^{\pm j\omega T}$$

*r θ Hvis T → 0
poler svære at skelne*

$$T = \frac{1}{f_{\text{sample}}}, \omega_s = 2\pi f_s \Rightarrow T = \frac{2\pi}{\omega_s} \Rightarrow$$

$$z = e^{\alpha T} e^{j\omega T} = e^{\alpha T} e^{j2\pi \frac{\omega}{\omega_s} T}$$

*ω = $\frac{\omega_s}{2}$ ⇒ vinkel → π
red 1/2 samplefrekvens*

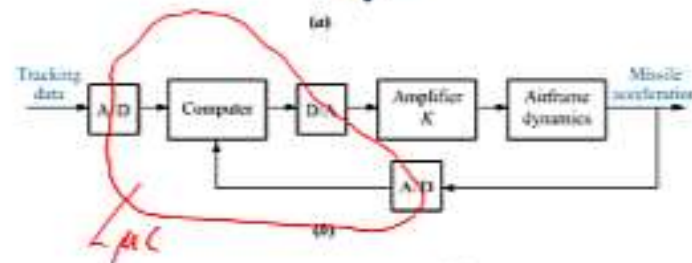
Eksempel Figure 13.14

Finding stability of a missile control system:

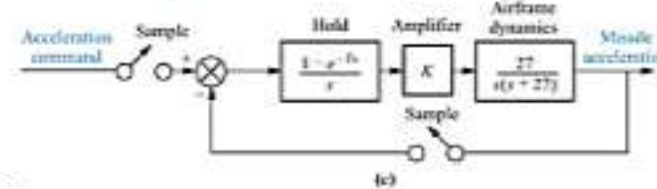
a. missile;



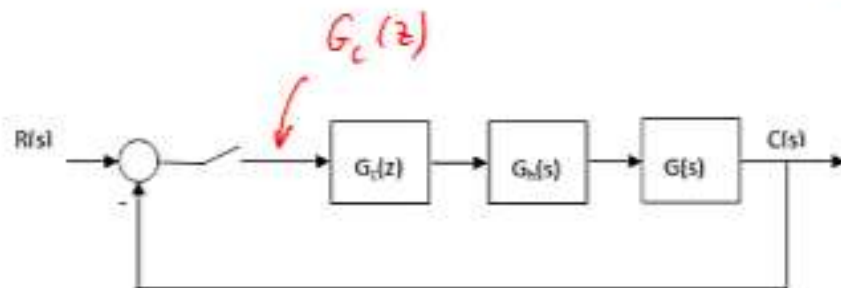
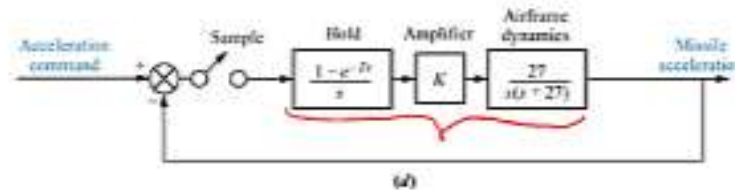
b. conceptual block diagram;



c. block diagram;



d. block diagram with equivalent single sampler



$$\frac{C(z)}{R(z)} = \frac{G_c(z) \cdot \overline{G_h G(z)}}{1 + G_c(z) \cdot \overline{G_h G(z)}}$$

$$\overline{G_h G(z)} = (1-z^{-1}) \mathcal{Z} \left\{ \frac{k \cdot 27}{s^2(s+27)} \right\}$$

Analyse i z-planet

$$\frac{C(z)}{R(z)} = \frac{G_c(z) \cdot \overline{G_h G(z)}}{1 + G_c(z) \cdot G_h G(z)}$$

Bodeplot $z = e^{j\omega T}$

$G_c(z) = K \frac{z-a}{z-b}$ mulige a og b indenfor
enhedscirklen

man er dimensionere i Bodeplot

DC-forstærkning? $G_c(z)|_{z=1}$

Vi dimensionerer i s-planet og transformerer så den dimensionerede regulator til z-planet, hvorfra programmet kan skrives.

$$G_c(s) \leftrightarrow G_c(z)$$

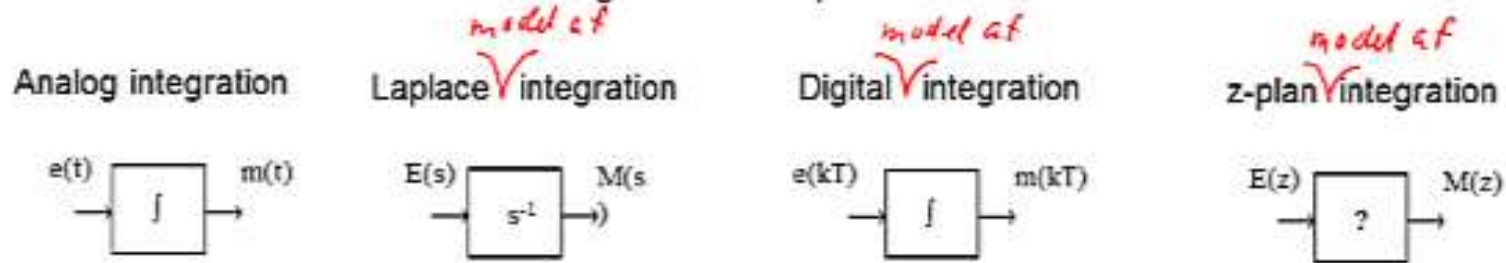
V.h.j.a. Bilineær transformation:

$$s = \frac{2(1-z^{-1})}{T(1+z^{-1})} \Rightarrow z = \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

en blandt mange
z-transformationer

Padé tilnærmelse
at $e^{sT} = z$

Bilineær transformation er en af de mange måder hvorpå man kan tilnærme transformationen $z = e^{sT}$



Den trapetzoide metode = Tustin

Integralet af $e(t)$ er til tiden $t=kT$: $m(kT)$

Tilnærmes til tiden $t = kT+T$ som:

$$m(kT+T) = m(kT) + [e(kT) + e(kT+T)] \cdot \frac{1}{2} \cdot T$$

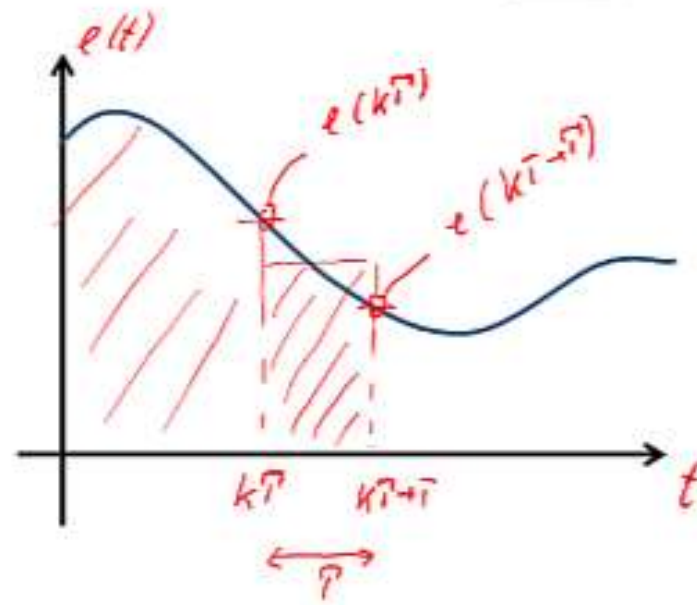
z-transf.

$$z \cdot M(z) = M(z) + [E(z) + z \cdot E(z)] \cdot \frac{1}{2} \cdot T$$

$$M(z)[z-1] = \frac{1}{2} \cdot T [z+1] E(z)$$

$$\frac{M(z)}{E(z)} = \frac{T}{2} \frac{z+1}{z-1} = \frac{1}{s} \Rightarrow s = \frac{2}{T} \frac{z-1}{z+1} = \text{bilineære transformationer}$$

ekvivalent med $\frac{1}{s}$



Eksempler på Z-transformationsmetoder

$$s = \frac{d}{dt}$$

$$m(kT) = \frac{e^{(k+1)T} - e^{kT}}{T}$$

$$\frac{T a}{1 - e^{-aT}}$$

bruges til
processer $G(s)$

TABLE 4-1 EQUIVALENT DISCRETE-TIME FILTERS FOR A CONTINUOUS-TIME FILTER $G(s) = a/(s + a)$

Mapping method	Mapping equation	Equivalent discrete-time filter for $G(s) = \frac{a}{s + a}$
Backward difference method	$s = \frac{1 - z^{-1}}{T}$	$G_D(z) = \frac{a}{\frac{1 - z^{-1}}{T} + a}$
Forward difference method	$s = \frac{1 - z^{-1}}{Tz^{-1}}$	This method is not recommended, because the discrete-time equivalent may become unstable.
Bilinear transformation method	$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$	$G_D(z) = \frac{a}{\frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} + a}$
Bilinear transformation method with frequency prewarping	$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$ $(\omega_s = \frac{2}{T} \tan \frac{\omega_0 T}{2})$	$G_D(z) = \frac{\tan \frac{aT}{2}}{\frac{1 - z^{-1}}{1 + z^{-1}} + \tan \frac{aT}{2}}$
Impulse-invariance method	$G_D(z) = T \mathcal{P} [G(s)]$	$G_D(z) = \frac{Ta}{1 - e^{-aT}z^{-1}}$
Step-invariance method	$G_D(z) = \mathcal{P} \left[\frac{1 - e^{-sT}}{s} G(s) \right]$	$G_D(z) = \frac{(1 - e^{-aT})z^{-1}}{1 - e^{-aT}z^{-1}}$
Matched pole-zero mapping method	A pole or zero at $s = -a$ is mapped to $z = e^{-aT}$. An infinite pole or zero is mapped to $z = -1$.	$G_D(z) = \frac{1 - e^{-aT}}{2} \frac{1 + z^{-1}}{1 - e^{-aT}z^{-1}}$

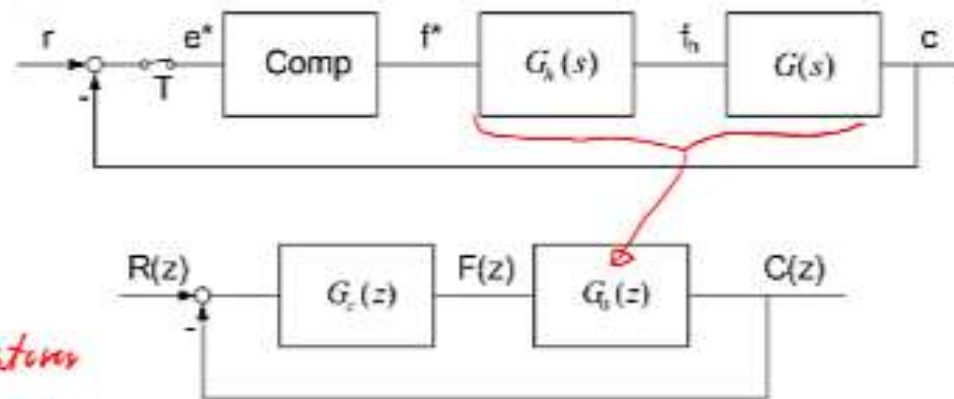
Cascade compensation via the s-plane

Vi vil på sædvanlig vis designe vores regulator, $G_c(s)$, ud fra Bodeplot for $G(s)$, åben-sløjfe overføringsfunktionen

$G_c(s)$ transformeres derefter til $G(z)$ med

$$s = \frac{2(1-z^{-1})}{T(1+z^{-1})} \Rightarrow z = \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

Matlab: $c2d(G_c, T, 'tustin')$ til regulatoren
 $c2d(G, T, 'zoh')$ til processen



I det analoge mangler:

Sample intervallet T medfører én forsinkelse af signalet på $T/2$

Altså tilføjelsen af overføringsfunktionen $e^{-sT/2}$ der giver et fasebidrag på: $-\omega T/2$ [rad] $\sim -\omega T 180/2\pi$ [°]



The sampling frequency is determined by Åström and Wittenmark:

$$\frac{0.15}{\omega_{\phi_m}} \leq T \leq \frac{0.5}{\omega_{\phi_m}} \Rightarrow 2\omega_{\phi_m} \leq f_s \leq 7\omega_{\phi_m}$$



$-4,3^\circ < \text{fasebidrag ved } \omega_{\phi_m} < -14,3^\circ$

Example 13.12

*20% OS
T_s = 1,1 s*

$$G(s) = \frac{1}{s(s+6)(s+10)}$$

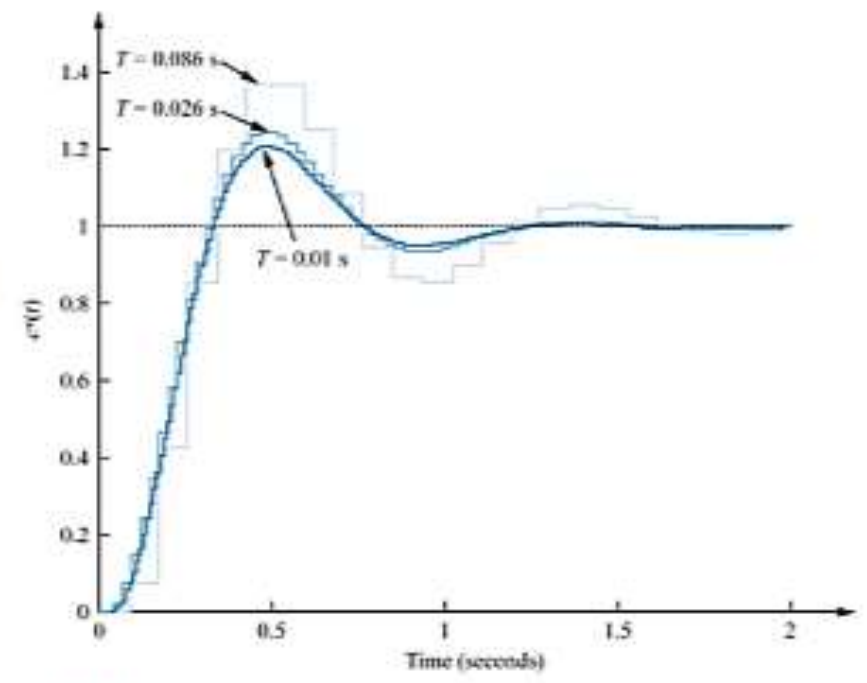
$$G_c(s) = \frac{1977(s+6)}{s+29,1} \quad \text{lag/lead?}$$

$$|G(j\omega)G_c(j\omega)| = 1 \Rightarrow \omega = \omega_{\varphi_m} = 5,8 \text{ rad/s} \Rightarrow$$

$$\frac{0,15}{\omega_{\varphi_m}} \leq T \leq \frac{0,5}{\omega_{\varphi_m}} = \frac{0,15}{5,8} \leq T \leq \frac{0,5}{5,8} = 0,026 \leq T \leq 0,086$$

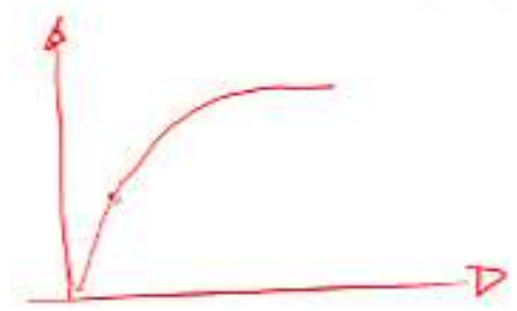
T = 0,01 ⇒

$$G_c(z) = \frac{1978z - 1674}{z - 0,746}$$



Iflg. Åström og Wittenmark: $0,026 < T < 0,086$

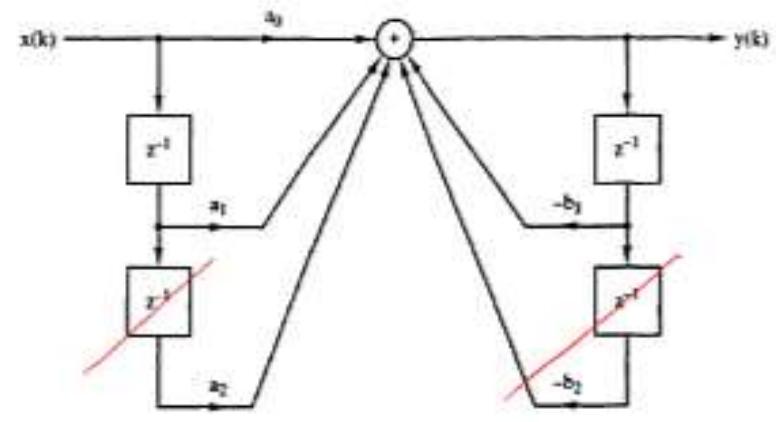
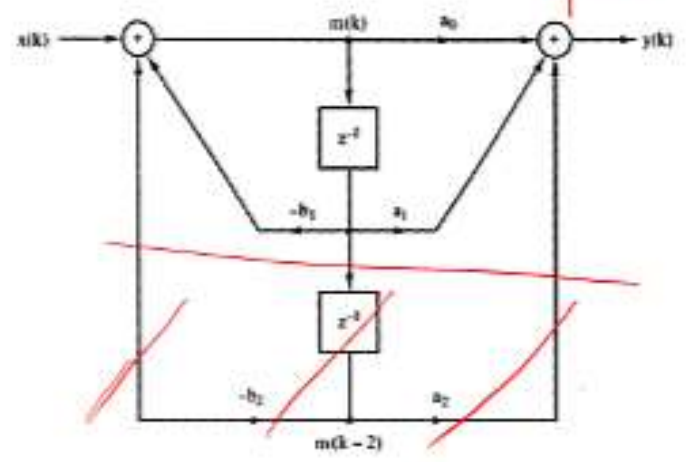
→ $-4,3^\circ < \text{fasebidrag ved } \omega_{\varphi_m} < -14,3^\circ$



Digital realising af regulatorer:

$$G_c(z) = \frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

$$G_c(z) = K \frac{z-4}{z-0}$$



Initialiser
 Afvent tidsflag

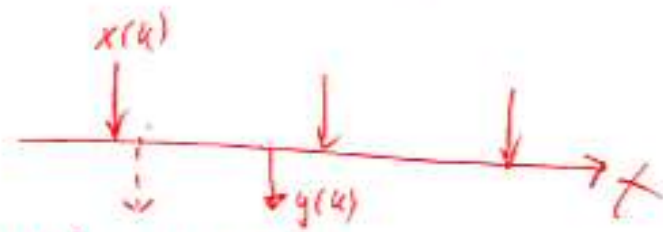
Indlæs x(k)

$$y(k) = (x(k) - b_1 m(k-1)) G_0 + G_1 m(k-1)$$

udlæs y(k)

$$m(k-1) = x(k) - b_1 m(k-1)$$

Gå til afvent tidsflag



Tilhørende opgaver

- Skill assesment exercise 13.3
- Øvelsesopgave kap. 13, stabilitet af samplede reguleringsystemer

Givet et samplet reguleringsystem med proportionalregulator:

forstærkningen K ,
0'te ordens holdeled,
reguleringsobjektet er $G(s) = s^{-1}$
enhedsfeedback,
samplingtiden er T .

1. Tegn et blokdiagram af systemet.
2. Bestem den største værdi af $K \cdot T$, for hvilken systemet er stabilt.
3. Bestem stepresponset for $K=10$ og $T=0.1$ sek.)
4. Verificer med Mallab, se ch13p5 i appendix B)

- Skill assessment Exercise 13.9

(including Matlab solution)

Skill-Assessment Exercise 13.9

Problem: In Example 11.3 a lead compensator was designed for a unity feedback system whose plant was

$$G(s) = \frac{100K}{s(s+36)(s+100)}$$

The design specifications were as follows: percent overshoot = 20%, peak time = 0.1 second, and $K_v = 40$. In order to meet the requirements, the design yielded $K = 1440$ and a lead compensator,

$$G_c(s) = 2.38 \frac{s+25.3}{s+60.2}$$

If the system is to be computer controlled, find the digital controller, $G_c(z)$.

$$T_{\text{sample}} = 0.001$$

- Forberedelse af øvelse 5 +6

Her ser vi både på virkningen af, at systemet er samplet og hvad der sker hvis, der direkte i processen også er en tidsforsinkelse

Her realiseret v.hj.a. en programstump i DSP'en