A15 & A16 – MAT B: 21/3 2017

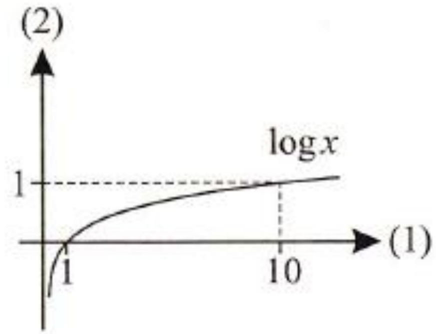
- Tilstedeværelsesregistrering
- Meddelelser. Spørgsmål? Lidt opgaver fra i går.
- Nyt stof:
  - Eksponential og logaritmefunktioner.
    - AB1 side 248-267.
    - BB>Supplerende note>Logaritme- og eksponentialfunktioner.
- Beviserne = Eksamensspøørgsmål
- Opgaver: BB>Opgaver>Logaritmefunktioner - opgaver 1 & 2 fortsat

8) Eksponential- og logaritmefunktioner

Der ønskes en redegørelse for sammenhængen mellem eksponential- og logaritmefunktioner samt omtale af deres differentialkvotienter. Hovedvægten kan lægges på $\log(x)$, $\ln(x)$, 10^x og e^x.

Logaritmeregnereglerne ønskes bevist ud fra potensregnereglerne.

Fordoblings- og halveringskonstant defineres, og der udledes formler for dem.



Grafen for logaritme-
funktionen med grundtal 10

(87) $\log x \to -\infty \quad \text{for} \quad x \to 0$

(88) $\log x \to \infty \quad \text{for} \quad x \to \infty$

(89) $y = \log x \quad \Leftrightarrow \quad x = 10^y$

(90) $\log 10 = 1$

(91) $\log(a \cdot b) = \log(a) + \log(b)$

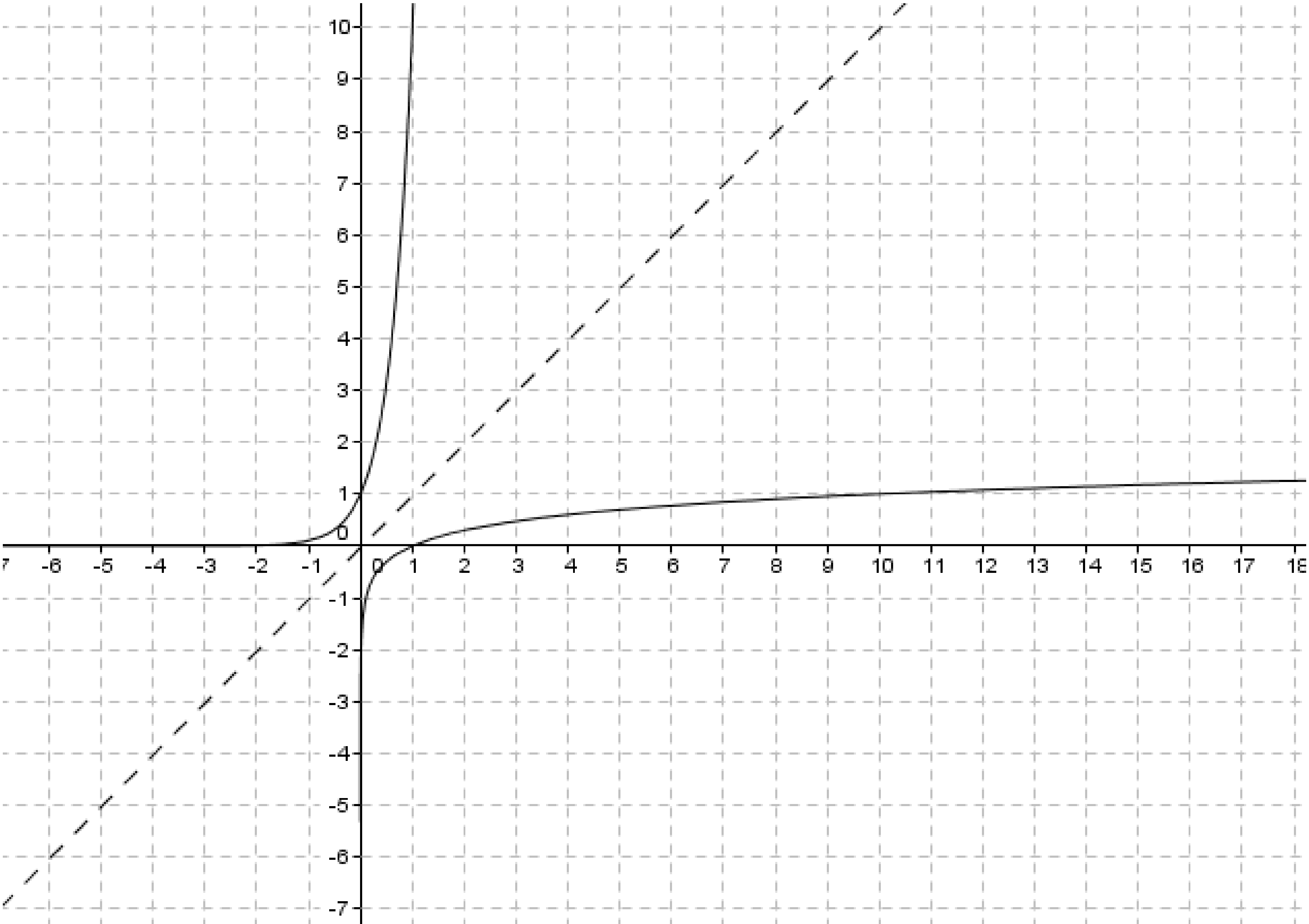
(92) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(93) $\log(a^r) = r \cdot \log(a)$

(99) $T_2 = \frac{\log 2}{\log a} = \frac{\ln 2}{\ln a}$

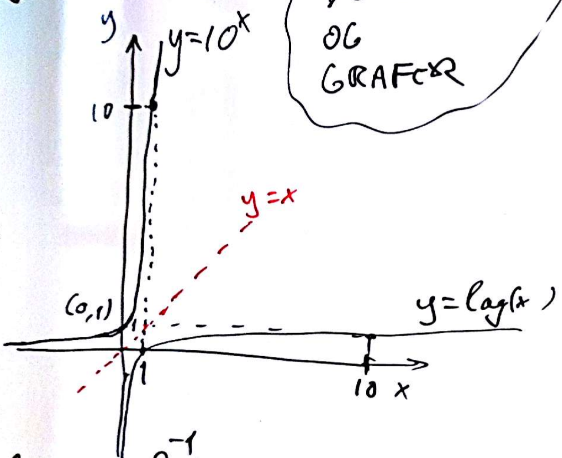
(105) $T_{\frac{1}{2}} = \frac{\log\left(\frac{1}{2}\right)}{\log(a)} = \frac{\ln\left(\frac{1}{2}\right)}{\ln(a)}$

(107) $a = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{x_2}{x_1}\right)} = \frac{\ln\left(\frac{y_2}{y_1}\right)}{\ln\left(\frac{x_2}{x_1}\right)}$


-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
10 9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7

$f(x) = 10^x$

DEFINITION OG GRAFER



$\log(x) = f^{-1}(x)$

$Dm(10^x) = \mathbb{R}$, $Vm(10^x) =]0;\infty[$

$Dm(\log) =]0;\infty[$, $Vm(\log) = \mathbb{R}$

$f(f^{-1}(x)) = x$

$$10^{\log(x)} = x \quad \boxed{x>0}$$

$f^{-1}(f(x)) = x$

$$\log(10^x) = x$$

$10^{r+s} = 10^r \cdot 10^s$

$10^{r-s} = \frac{10^r}{10^s}$

$10^{r \cdot s} = (10^s)^r$

REGNEREGLER

$\log(a \cdot b) = \log(a) + \log(b)$ ✓

$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ ✓

$\log(a^r) = r\log(a)$ ✓

BEVIS

$\log(a) + \log(b) =$

$\log\left(10^{\log(a)+\log(b)}\right) =$

$\log\left(10^{\log(a)} \cdot 10^{\log(b)}\right) =$

$\log(a \cdot b)$ ✓

$$\log(a) - \log(b) =$$

$$\log\left(10^{\log(a) - \log(b)}\right) =$$

$$\log\left(\frac{10^{\log(a)}}{10^{\log(b)}}\right) =$$

$$\log\left(\frac{a}{b}\right)$$

$$r \log(a) =$$

$$\log\left(10^{r \log(a)}\right)$$

$$\log\left(\left(10^{\log(a)}\right)^{r}\right)$$

$$\log\left(a^{r}\right)$$

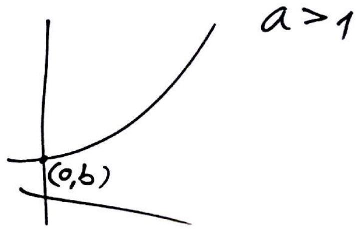
EKSPONENTIEL VÆKST

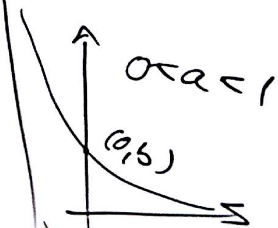
$$f(x) = b \cdot a^{x}$$

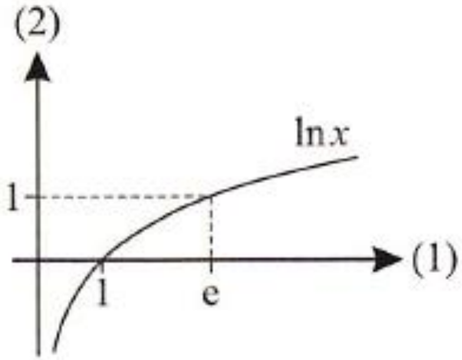
$a > 1$

$b > 0$

$0 < a < 1$







Grafen for den naturlige logaritmefunktion

(80) $\ln x \to -\infty \quad \text{for} \quad x \to 0$

(81) $\ln x \to \infty \quad \text{for} \quad x \to \infty$

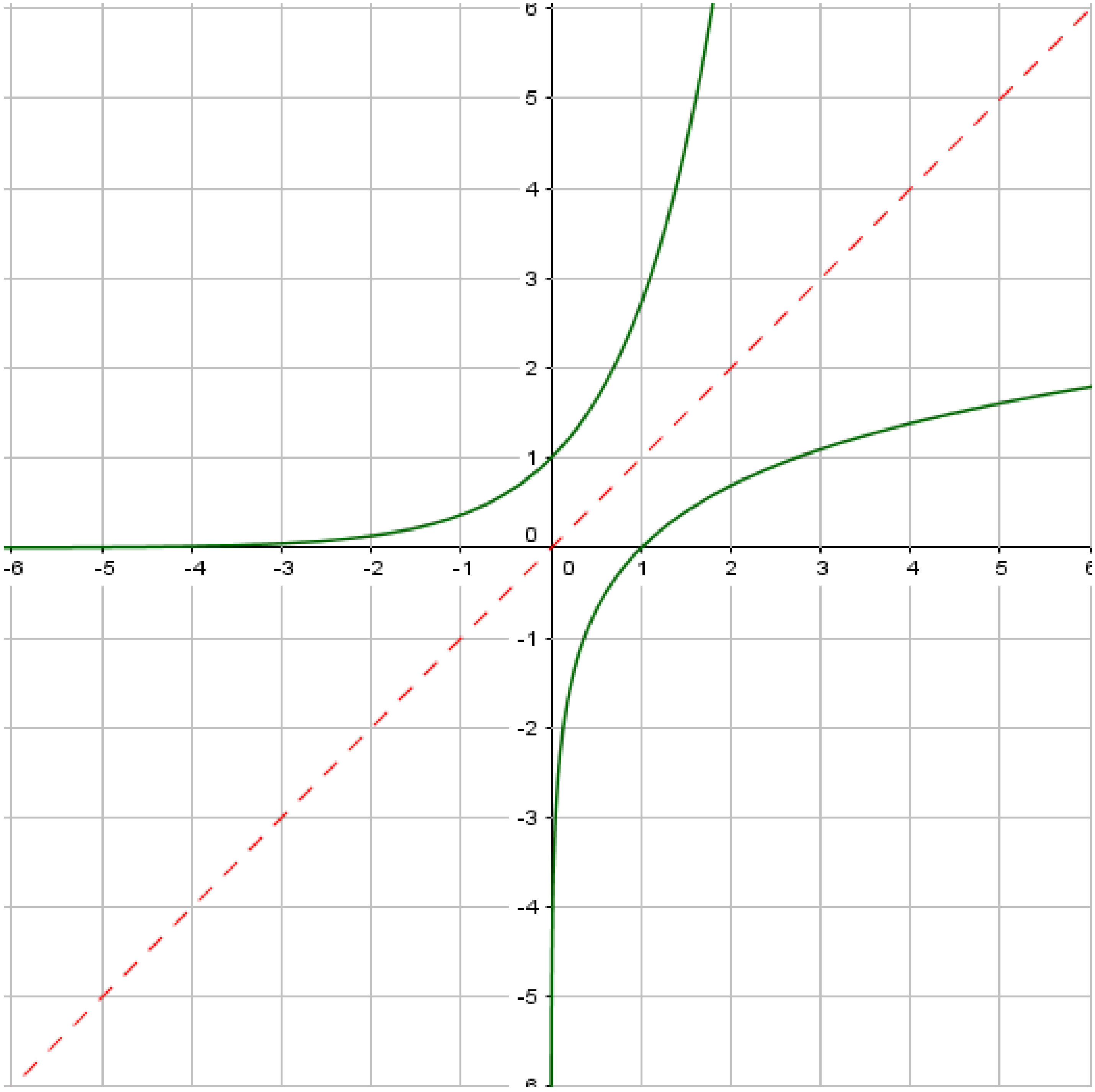
(82) $y = \ln x \quad \Leftrightarrow \quad x = e^y$

(83) $\ln e = 1$

(84) $\ln(a \cdot b) = \ln(a) + \ln(b)$

(85) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

(86) $\ln(a^r) = r \cdot \ln(a)$



-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
-1
-2
-3
-4
-5

(98) $$T_2 = x_2 - x_1$$

(99) $$T_2 = \frac{\log 2}{\log a} = \frac{\ln 2}{\ln a} = \frac{\ln 2}{k}$$

(104) $$T_{\frac{1}{2}} = x_2 - x_1$$

(105) $$T_{\frac{1}{2}} = \frac{\log\left(\frac{1}{2}\right)}{\log(a)} = \frac{\ln\left(\frac{1}{2}\right)}{\ln(a)} = \frac{\ln\left(\frac{1}{2}\right)}{-k} = \frac{\ln 2}{k}$$

Og nu til noget helt andet…..

Potensfunktion (106) $f(x) = b \cdot x^a$

(107) $$a = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{x_2}{x_1}\right)} = \frac{\ln\left(\frac{y_2}{y_1}\right)}{\ln\left(\frac{x_2}{x_1}\right)}$$