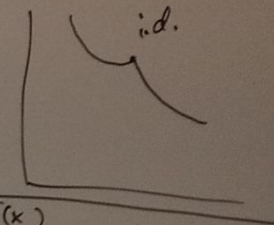
Tavler fra mandag 26/10-2015
Differentiationsregler - bevis

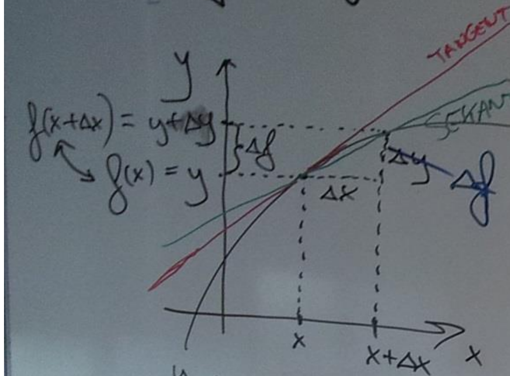
$(f(x)+g(x))' = f'(x) + g'(x)$ ✓

$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$



$a_t = f'(x)$

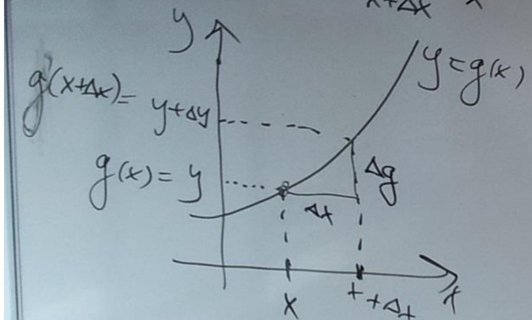
DIFFERENSKVOTIENT



$a_s = \frac{\Delta y}{\Delta x} \to a_t = f'(x)$ for $\Delta x \to 0$

$\Delta f = \underbrace{f(x+\Delta x)} - f(x)$

$\Delta f + f(x) = f(x+\Delta x)$



$\frac{\Delta y}{\Delta x} = ?$ $\quad S(x) = f(x) + g(x)$

$\Delta y = \Delta S = S(x+\Delta x) - S(x) = f(x+\Delta x) + g(x+\Delta x) - (f(x) + g(x))$

$= \underbrace{f(x+\Delta x) - f(x)} + g(x+\Delta x) - g(x)$

$\Delta S = \Delta f + \Delta g$

$\frac{\Delta S}{\Delta x} = \frac{\Delta f + \Delta g}{\Delta x} = \frac{\Delta f}{\Delta x} + \frac{\Delta g}{\Delta x} \longrightarrow f'(x) + g'(x)$ for $\Delta x \to 0$

$$p(x) = f(x)\cdot g(x)$$

$$\Delta p = p(x+\Delta x) - p(x) = f(x+\Delta x)\cdot g(x+\Delta x) - f(x)\cdot g(x)$$

$$= (\Delta f + f(x))\cdot(\Delta g + g(x)) - f(x)\cdot g(x)$$

$$= \Delta f\cdot\Delta g + \Delta f\cdot g(x) + f(x)\cdot\Delta g + \cancel{f(x)\cdot g(x)} - \cancel{f(x)\cdot g(x)}$$

$$\frac{\Delta p}{\Delta x} = \frac{\Delta f\cdot\Delta g + \Delta f\cdot g(x) + f(x)\cdot\Delta g}{\Delta x} = \frac{\Delta f}{\Delta x}\cdot\Delta g + \frac{\Delta f}{\Delta x}\cdot g(x) + f(x)\cdot\frac{\Delta g}{\Delta x}$$

$$\to f'(x)\cdot 0 + f'(x)\cdot g(x) + f(x)\cdot g'(x) \quad \text{for } \Delta x \to 0$$