TALAT Training in Aluminium Application Technologies

TALAT Lecture 2301

Design of Members

125 pages, 92 figures

Advanced Level

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Objectives:

– To give background to calculation methods for aluminium members in order to understand the specific behaviour of statically loaded aluminium alloy structures.

Prerequisites:

– Basic structural mechanics and design philosophy
– Structural aluminium alloys and product forms

Note:

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2301 Design of Members

Contents

1 General

The objective of this part of TALAT is to give background to the design methods and recommendations in ENV 1999-1-1 Design of aluminium structures Part 1-1: General rules (Eurocode 9) in order to understand the specific behaviour of static loaded aluminium structures. The text is supplemented by a number of design examples.

1.01 Scope

This chapter concern aluminium plate structures and extrusions primarily applicable to aluminium structures in buildings, civil and structural engineering works such as bridges, hydraulic and offshore structures and temporary structures, e.g. erection and building scaffolds, masts, cable- and light poles.

1.02 Symbols

Symbols

The symbols used in Eurocode 9 are mostly used in this document as well. In order to facilitate the reading the most common symbols are given here.

A_{eff}	Effective area
A_{gr}	Gross area
A_{net}	Net area
b	Width
b_{haz}	Width of heat affected zone
c	Distance; Outstand
d	Diameter; Depth
E	Modulus of elasticity
f_o	Characteristic strength for yielding
f_a	Characteristic ultimate strength for the local capacity in net section
$f_{0,2}$	0,2 proof strength
f_u	Ultimate strength
f_{haz}	Characteristic strength in heat-affected zone
G	Shear modulus
I_{eff}	Second moment of area of effective cross section (I_e with further subscript)
I_{gr}	Second moment of area of gross cross section
I_{net}	Second moment of area of net cross section
I_{fic}	Fictive second moment of area for calculation of deflections
L	length, span, system length
l_c	Effective (buckling) length
M_{el}	Elastic bending moment
M_{pl}	Plastic bending moment
M_{Ed}	Bending moment (action), design value
M_{Rd}	Bending moment resistance, design value
N_{Ed}	Axial force (action), design value
N_{Rd}	Axial force resistance, design value
q	Distributed load
r	Radius
t	Thickness

t_{eff}	Effective thickness (t_e with further subscript)
W_{el}	Elastic section modulus
W_{pl}	Plastic section modulus
W_{eff}	Section modulus of effective cross section
α	Shape factor
β	Slenderness ratio of a cross section element
γ_{M1}	Partial safety factor for the resistance due to overall yielding
γ_{M2}	Partial safety factor for the resistance of net section
ε	Strain
λ_c	Slenderness parameter ($\bar{\lambda}$ in Eurocode 9)
l_c/i	Slenderness ratio (λ in Eurocode 9)
ρ_c	Reduction factor for local buckling
ρ_{haz}	HAZ softening factor
σ	Stress
χ	Reduction factor for flexural buckling
χ_{LT}	Reduction factor for lateral-torsional buckling
ψ	Stress ratio
ψ	Factor defining representative values of variable actions
ψ_c	Exponent in interaction formula
ξ_{yc}	Exponent in interaction formula
ξ_{zc}	Exponent in interaction formula
η_c	Exponent in interaction formula
ω_x	HAZ softening factor in interaction expressions

S.I. units

The following S.I. units are used

- Forces and loads	kN, kN/m, kN/m^2
- unit mass	kg/m^3
- unit weight	kN/m^3
- stresses and strength	N/mm^2 (=MN/m^2 or MPa)
- moments	kNm

1.03 Safety and serviceability

The design philosophy and the design procedure are discussed in Aluminium Design, part 4.1. In this chapter a short presentation of the partial coefficient method is given. Design values of strength of aluminium alloys are given in sub clause 2, Design basis.

In all modern codes of practice structural safety is established by the application of the partial safety coefficients to the loads (or 'actions') and to the strength (or 'resistance') of components of the structure. The new Eurocodes for the design and execution of buildings and civil engineering structures use a limit state design philosophy defined in Eurocode 1. (Common unified rules for different types of construction and material).

The partial safety coefficients for actions (γ_F) depend on an accepted degree of reliability, which is recognised as a national responsibility within the European Community. The probability of severe loading actions occurring simultaneously can be found analytically, if enough statistical information exists, and this is taken into account by the introduction of a second coefficient ψ. The design value of the action effects (when the effects are unfavourable) is then found by taking values of γ_F dependent on the type of loading and values for ψ that take account of the chances of simultaneous loading. A value of γ_F of 1,15 is suggested for permanent loads, such as the dead load of bridge girders, and 1,5 for variable loads such as traffic loads or wind loading. The loading actions on members are found by an elastic analysis of the structure, using the full cross-sectional properties of the members.

The partial safety coefficients for actions takes account of the possibility of unforeseen deviations of the actions from their representative values, of uncertainty in the calculation model for describing physical phenomena, and uncertainty in the stochastic model for deriving characteristic codes.

The partial safety coefficient for material properties (γ_M) reflects a common understanding of the characteristic values of material properties, the provision of recognised standards of workmanship and control, and resistance formulae based on minimal accepted values. The value given to γ_M accounts for the possibility of unfavourable deviations of material properties from their characteristic values, uncertainties in the relation between material properties in the structure and in test specimens, and uncertainties associated with the mechanical model for the assessment of the resistance capacity.

A further coefficient, γ_n, is specified in some codes, and this can be introduced to take account of the consequences of failure in the equation linking factored actions with factored resistance. It is often incorporated in γ_M. It recognises that there is a choice of reliability for classes of structures and events that take account of the risk to human life, the economic loss in the event of failure, and the cost and effort required to reduce the risk. Typical values in recent European codes of practice for aluminium are $\gamma_M \cong \gamma_n$ = 1,2 and 1,3, on the assumption that properties of materials are represented by their characteristic values.

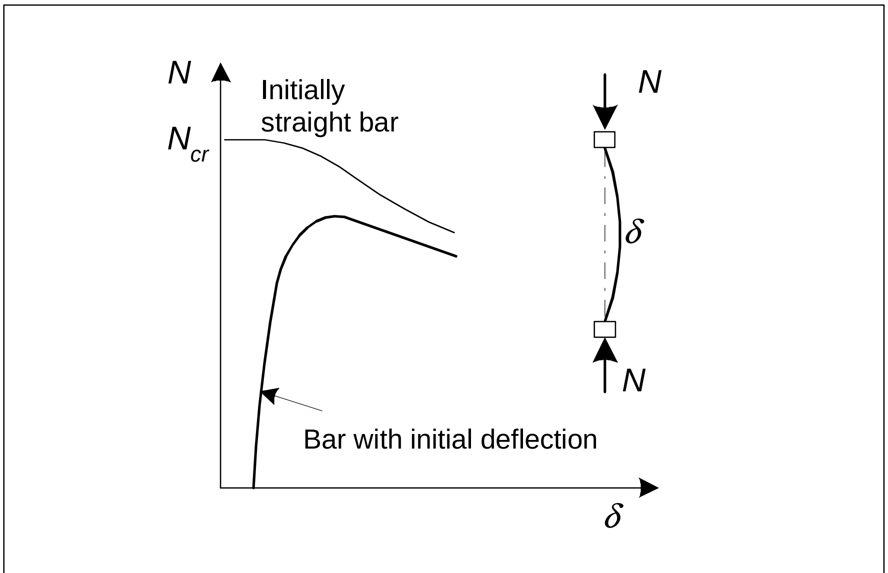
The ultimate limit states defined by the use of the above factors refer to failure of members or connections by rupture or excessive deformation, transformation of the structure into a mechanism, failure under repeated loading (fatigue) and the loss of equilibrium of the structure as a rigid body.

Serviceability limit states, according to most definitions, correspond to a loss of utility beyond which service conditions are no longer met. They may correspond to unacceptable deformations or deflections, unacceptable vibrations, the loss of the ability to support load-retaining structures, and unacceptable cracking or corrosion. Because certain aluminium alloys in the non-heat-treated condition, or in the work-hardened condition, do not have a sharply defined 'knee' to the stress/strain curve, it is sometimes possible for unacceptable permanent deformation to occur under nominal or working loads. The same may be true for alloys that have a substantial amount of welding during fabrication.

1.04 Design with regards to instability

Extruded and welded members are never totally perfect. They possess a number of imperfections. Residual stresses, heat-affected zones and other variation of material properties and the Baushinger effect are dealt with in section 1.06 and 1.07. Other types of imperfections, for example initial curvature and deviation of cross sectional dimensions, are dealt with in section 1.05. The stress-strain relationship of aluminium is dealt with in section 1.08.

It is of great importance to take the influence of imperfections into consideration, especially for different types of instability phenomena, e.g. flexural buckling, lateral-torsional buckling and plate buckling. To illustrate this, consider the case of flexural buckling for a bar subjected to an axial load, cf. **Figure 2301.01.01**. In the past, the compressive force capacity was calculated with Euler's buckling formula. This formula is valid for a perfectly straight, elastic bar without imperfections. However, in reality, such a bar contains a number of imperfections that reduce the strength. In **Figure 2301.01.01** the behaviour of an idealised Euler column is compared to that of a real column.



TALAT Training in Aluminium Application Technologies	Comparison between buckling behavior of an idealized Euler bar and of a real bar with imperfections	2301.01.01

It is possible, in the age of computers, to create calculation models that can, with great detail, simulate the actual behaviour, but under one condition. Every imperfection of the beam must be known and correctly modelled and taken into consideration. Residual stresses and variation in material properties have little influence on the behaviour of extruded members. On the other hand, the first two of these imperfections can have great effect on welded members.

Welding effects the member by creating residual stresses and reduction in strength of the material in the heat affected zones.

1.05 Geometrical imperfections

1.051 Extruded profiles

Initial curvature

The deviation of cross section dimensions and member length is often very small for extruded members. The effect is however not negligible if there is risk for instability, e.g. flexural buckling and lateral-torsional buckling.

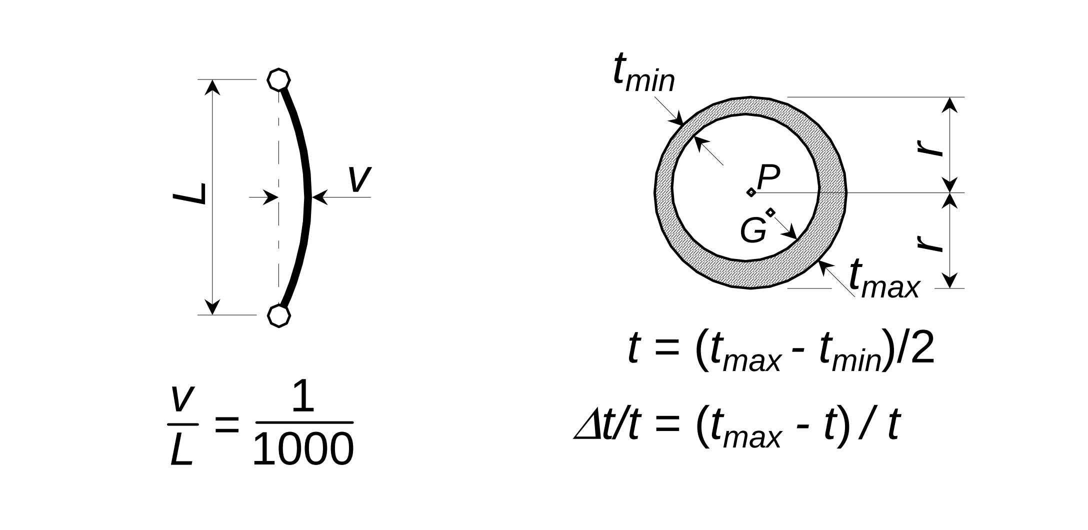
A systematic analysis carried out on extrusions from several European countries showed that the initial curvature was approximately L/2000. In national specifications, initial curvature is usually presumed to lie between L/500 and L/1000. It is common to use the same value as for rolled steel sections. The ECCS (European Convention for Constructional Steelwork) recommends that v_0 is taken as L/1000 when calculating buckling curves for extruded profiles, cf. **Figure 2301.01.02**. This value may seem to be far on the safe side. However, another geometrical imperfection is considered within this value; deviation of sectional dimensions.

Deviation of cross sectional dimensions

Deviation of width for sectional parts is usually lower than 1 percent. The thickness can vary 5 percent and for parts thinner than 5 mm the deviation can be up to 10 percent. The thickness of hollow extrusions can have large variations. This is due to the extrusion process. Even a perfect die causes deviation of thickness in a hollow extrusion. This is enhanced in extrusions with small cross sections, cf. **Figure 2301.01.02**.

The effect of this imperfection is that a centric compressive load actually has a certain eccentricity. The point of load introduction, in this case, does not coincide with the centre of gravity for the cross section.

For extruded profiles and welded profiles, measurements show that this eccentricity, e, is less than L/1600. Together with the initial curvature of L/2000, this explains why initial curvature in national regulations is considered to be less than L/1000.



TALAT Training in Aluminium Application Technologies	Definition of initial curvature and eccentricity	2301.01.02

Initial buckles

Flat parts in extruded profiles show very small initial buckles. This is of two reasons; the first is that it is difficult to produce extrusions with sectional parts so slender that initial buckles can develop. The second reason is that the traction process that follows extrusion reduces any initial buckles.

1.052 Welded profiles

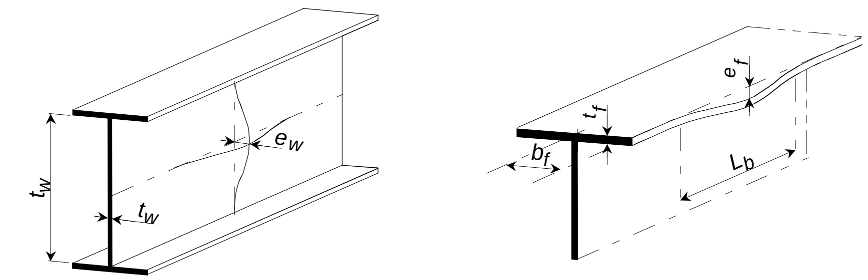
The influence of residual stresses and the strength reduction in the heat-affected zone, are dealt with in section 1.06 and 1.07. In this section, initial curvature and variation in sectional dimensions are discussed.

Measurement of welded T-sections and box sections show that the initial curvature is always less than L/1300, i.e. always greater than for extruded profiles.

The cross section measurement is within the same tolerance limits as for extruded profiles. One important imperfection on welded I-sections, where the web is welded directly to the flanges with fillet welds, is that the web plate often is a little eccentric. Measurements on such profiles showed eccentricities not greater than L/1600. Conclusively, this means that the same initial curvature can be used for welded profiles as for extruded profiles, when determining buckling curve, i.e. L/1000.

Initial buckles in flat cross section elements

Initial buckles in welded beams (flanges and webs) cannot be avoided. The following tolerances are recommended, if smaller tolerances are not necessary for aesthetic or other reasons.



TALAT Training in Aluminium Application Technologies | Tolerated divergence from flatness of web and flanges | 2301.01.03

Tolerated buckles in the web are given in the following expressions. The limit is applied to each panels of the web with horizontal stiffeners.

$$e_w < \frac{b_w}{200} \quad \text{when} \quad \frac{b_w}{t_w} \leq 50$$

$$e_w < \frac{b_w^2}{10000\, t_w} \quad \text{when} \quad 50 < \frac{b_w}{t_w} < 125 \qquad (1.01)$$

$$e_w < \frac{b_w}{80} \quad \text{when} \quad \frac{b_w}{t_w} \geq 125$$

The largest tolerable deflections for an outstand element in compression, i.e. the compression flange for an I- ,U- or Z- cross section, are given in the following expressions.

$$e_f < \frac{L_b}{250} \quad \text{when} \quad \frac{b_f}{t_f} \leq 10$$

$$e_f < \frac{L_b\, b_w}{2500\, t_f} \quad \text{when} \quad \frac{b_f}{t_f} > 10 \qquad (1.02)$$

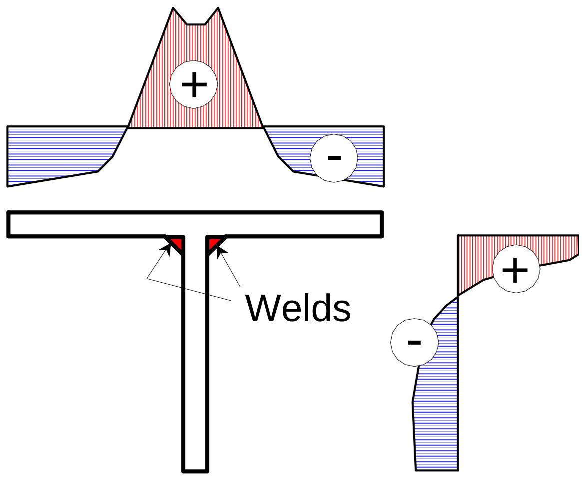
1.06 Residual stresses and variability in material properties

Extruded and welded profiles contain imperfections of different kinds. In this section, residual stresses, variability in material properties and the Bauschinger effect are dealt with. Other kinds of imperfections, e.g. initial curvature and variability in dimensions of the cross section are dealt with in section 1.5.

1.061 Residual stresses

If a part of a member undergoes nonuniform, plastic deformation stresses arise within the elastic area. The sum of negative and positive stresses is always zero, if there are no external forces. The inhomogeneous deformation field which generates residual stress is caused by thermal processes such as cooling after extrusion and welding, mechanical processes such as cold rolling and straightening by means of traction.

For a welded T-profile the residual stresses may be formed as follows:
The weld is very warm in the beginning. The zone next to the weld is also very warm. When the material cools down, the weld shrinks because of differences in density between the hard and the soft material. Further, the weld will shrink because of the thermal diffusion factor. The surrounding cold and stiff metal prevent this shrinking. This part of the cross section is subject to compressive stresses while the area closest to the weld string is loaded with tensile stresses. See **Figure 2301.01.04**.



TALAT Training in Aluminium Application Technologies	Example of residual stresses in an welded T-profile	2301.01.04

How to measure residual stress

The most common method is the destructive method, which is based upon the technique of cutting the specimen in a number of strips. The residual stresses are calculated from measurements on each strip.

There are two methods of measuring. The first is to measure the length of the strip before and after the cutting it from the section. If Young's modulus is known, it is easy to apply Hooke's law and determine the residual stress. The second method is to mount electrical resistance strain gauges on the strips and determine the residual stresses by applying Hooke's law. The last method is that which is most commonly used today. Note that Hooke's law can be applied since residual stress is essentially an elastic process. With the methods stated here only longitudinal residual stresses are determined. However, these are of most interest from a structural point of view.

Residual stress in extruded profiles

Mazzolani [see TALAT 2600] has shown results from a number of experiments where residual stresses are determined for different types of profiles. These consist of different alloys and were manufactured by various processes.

Here, the results from experiments on I-profiles are reviewed.

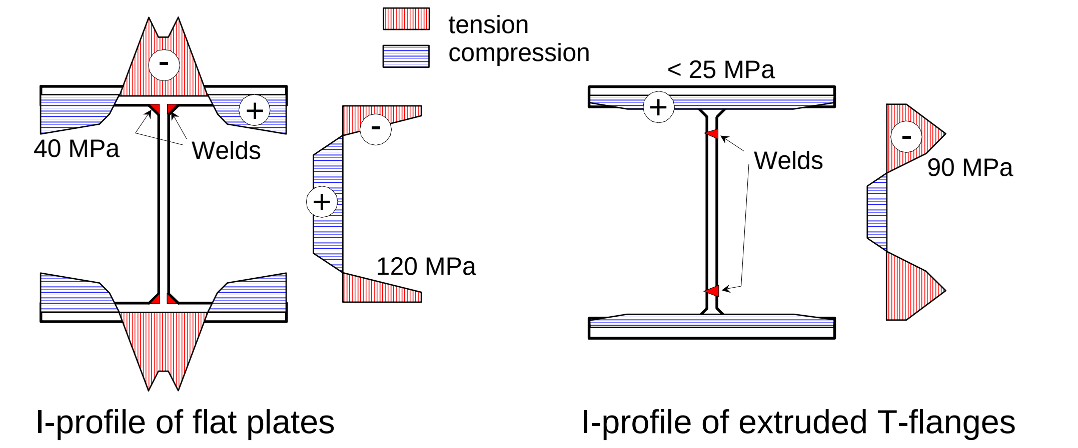
Experiments conducted on I-profiles consisting of different alloys show that the residual stresses are randomly distributed over a cross section. It seems there is no simple rule for stress distribution as there is for rolled steel sections. Residual stresses are low, the compressive stresses almost never exceed 20 MPa and tensile stresses are much lower. These values are measured on the surface of the profiles. At the centre of the material the values are probably lower since residual stresses usually change sign from one side to the other. Different alloys do not affect the intensity and distribution of residual stresses.

The residual stresses have a negligible effect on the load-bearing capacity.

Residual stresses in welded profiles

In contrast to what has been said for extruded profiles, residual stresses cannot be neglected in welded profiles. The welding produces a concentrated heat input, which causes the remaining stresses (see above). Large tensile stresses in connection to the web and balancing compression stresses in other parts are characteristic.

Figure 2301.01.04 showed an example of the results of an experiment where two aluminium plates were welded together with butt welds. The plates were of different sizes, and alloys. One was of a heat-treated alloy and one of a non heat-treated alloy. The differences in the results were very small. The maximum tensile stress was 90 - 100 N/mm^2 and the maximum compression stress was 30 - 40 N/mm^2. The lower values correspond to the heat-treated alloy.

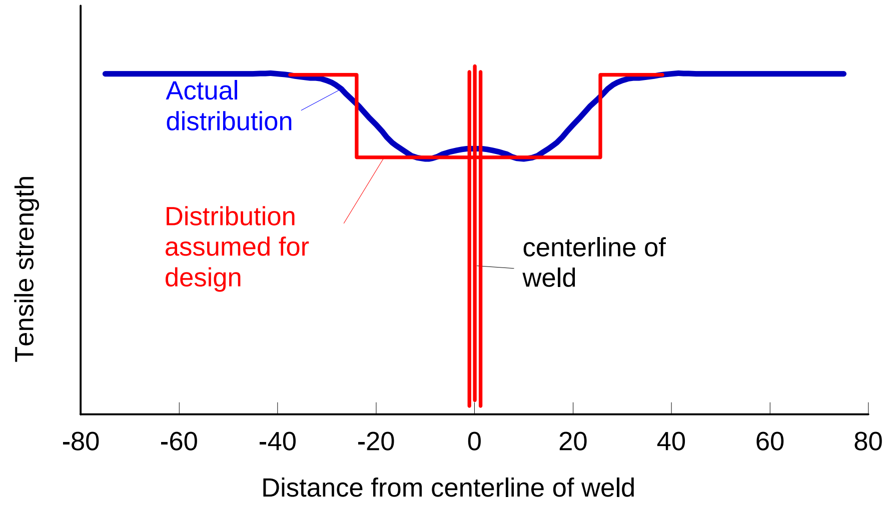


TALAT Training in Aluminium Application Technologies | Typical residual stresses in welded I-profiles | 2301.01.05

Typical residual stress distributions for two different I-profiles are shown in **Figure 2301.01.05**. The highest values of tensile stress were in both cases 140 N/mm^2. The greatest value of compressive stress was 50 N/mm^2 except for the web of the specimen to the right in **Figure 2301.01.05** where compressive stresses reached 100 N/mm^2. One conclusion of these two experiments is that by welding together extruded profiles one can place the welds within the area where they cause a minimum of strength reduction.

These experiments also show that the residual stresses in relation to the yield limit of the material are much lower for aluminium profiles than for corresponding steel profiles. In the experiments, the residual tensile stresses were less than 60% of the 0,2 percent proof stress. For steel, residual stresses may be greater than the yield stress of the material. The residual compressive stresses are typically 20% for aluminium and 70% for steel. Therefore, considering residual stresses, it is more favourable to weld aluminium than it is to weld steel. However, the strength of the material is reduced up to 50% in the zone around the weld for aluminium. This counterbalances the effect of the residual stress distribution.

Often the continuous curves of the residual stresses are replaced by equivalent block curves as those in **Figure 2301.01.06**. These curves are easier and faster to use in numerical models.



TALAT Training in Aluminium Application Technologies	Actual and simplified stress curve near a welded line	2301.01.06

1.062 Inhomogeneous distribution of mechanical properties

The mechanical properties such as Young's modulus, $f_{0,2}$, differ very little over the cross section, not more than a few percent. The difference is negligible when determining the resistance.

1.063 Bauschinger effect

If a specimen is loaded in tension and after that loaded in compression, the elastic limit is lower than for a specimen only loaded in compression.

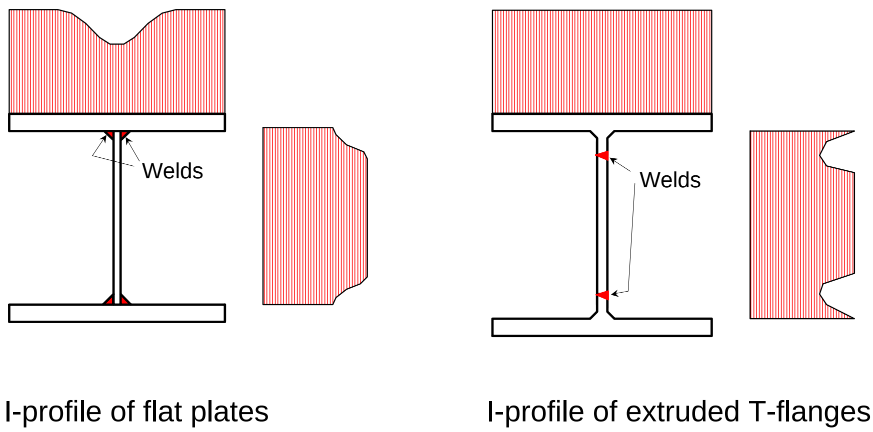
To eliminate initial curvature extruded profiles are straightened by traction. At the same time minor residual stresses are reduced. Initial curvature is an imperfection that reduces the strength, especially the compressive strength. The straightening also causes plastic deformation, which, according to the Bauschinger effect, reduces the resistance.

Normally the Bauschinger effect is neglected in national regulations. The reason is that the Bauschinger effect is more or less counterbalanced by the effect of the loss of residual stress when straightening extruded profiles. Furthermore the design methods (buckling curves etc.) have been calibrated with tests on specimens including the Bauschinger effect.

1.07 Heat affected zones

Two groups of welded profiles are distinguished: those consisting of heat-treated alloys and those consisting of non heat-treated alloys. The non heat-treated group is hardly affected by welding. The heat-treated group loses quiet an amount of strength in the heated affected zone close to the weld. The proof stress decreases up to 40-50 %. The reason for the phenomenon is that the heat-treated alloy is heated at the weld. The crystal structure is changed and the material loses its strength. **Figure 2301.01.06 a** shows, in principle, the distribution of material strength in the heat-affected zone.

The elastic limit, the strength at rupture and the elongation at rupture are influenced by welding when joining flat plates to an I-profile. The moment capacity is greatly reduced for such a beam. One solution is to place the weld in an area where the effect of welding is small on the bending strength. This can be achieved by welding together extruded profiles. See **Figure 2301.01.07**.

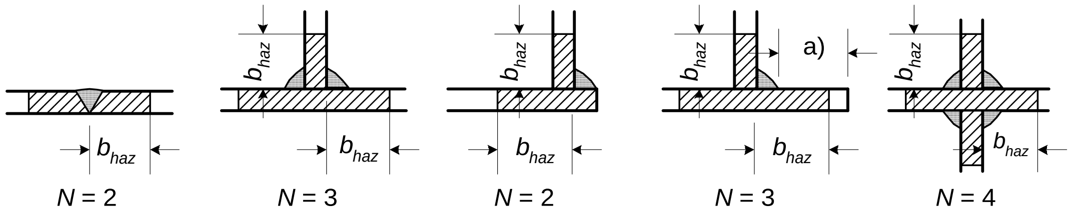


TALAT alu Training in Aluminium Application Technologies | Strength reduction in welded I-profiles | 2301.01.07

Influence of heat-affected zones

In ultimate limit state design the factored characteristic loads must be shown to be less than or equal to the calculated resistance of the structure or component divided by the material factor. In calculating the resistance of welded aluminium components, however, a problem occurs with the strong heat-treated alloys. The effect of the temperature generated by the welding process is to disrupt the heat treatment and produce softened zones in the vicinity of welds. This softening is a significant factor in 6xxx and 7xxx series alloys, and in 5xxx series alloys in a work-hardened temper. It can have a noticeable effect on the ultimate strength of the welded component and must be allowed for in design.

The extent of the HAZ is affected by the metal temperature when welding begins and by the build-up of temperature in multi-passes. When neighbouring parallel welds are laid simultaneously the extent of their combined HAZ increases. For thicker material the extent of the HAZ measured radial from all points along the edge of a weld was found to be proportional to $\sqrt{A_W/N}$, where A_W is the total section area of the weld deposit per pass and N is the number of heat flow paths adjacent to the weld. The extent was increased if temperature build-up was allowed to take place between passes.



a) If this distance is less than $3b_{haz}$ assume that the HAZ extends to the full width of the flanges

TALAT Training in Aluminium Application Technologies	**The extent of the heat-affected zones**	2301.01.08

For thinner material the extent of the HAZ measured radial from the centre-line or root of a weld was found to be proportional to A_w/N and inversely proportional to the mean thickness of the heat flow paths. The extent was increased if temperature build-up occurred between passes, although for thin material multi-pass welding is less likely to be required.

When a weld is located too near the free edge of an outstand the dispersal of heat is less effective, and the degree of softening of the alloy is increased. The effect depends on the ratio between the distance and the weld to the free edge and the extent of the HAZ calculated as if there were no free edge effect.
The conclusions of resent research have been incorporated into Eurocode 9, see table 1.01 and figures 1.08 and 1.09. The HAZ is assumed to extend a distance b_{haz} in any direction from a weld, measured as follows (see **Figure 2301.01.08**).

a) transversely from the centre line of an in-line butt weld
b) transversely from the point of intersection of the welded surfaces at fillet welds
c) transversely from the point of intersection of the welded surfaces at butt welds used in corner, tee or cruciform joints.
d) in any radial direction from the end of a weld.

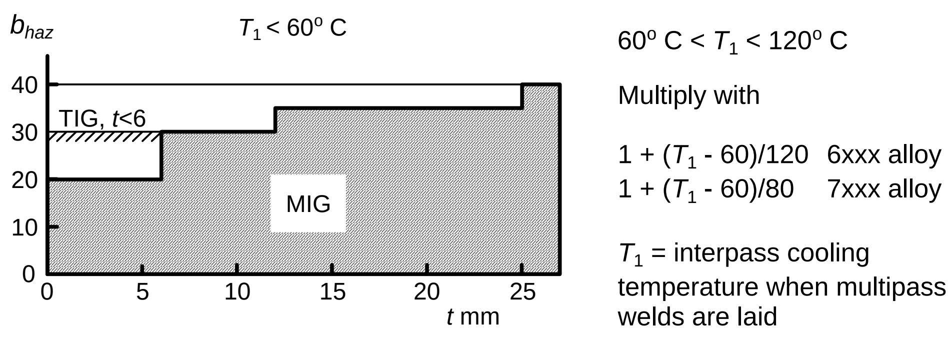
For thickness > 12 mm there may be a temperature effect, because interpass cooling may exceed 60°C unless there is strict quality control. This will increase the width of the heat-affected zone, see **Figure 2301.01.09**.

The values of b_{haz} from **Figure 2301.01.08** apply to in-line butt welds (two valid heat paths = two plates welded together) or to fillet welds at T-junctions (three valid heat paths). If the junctions between elements are fillet welded, but have different numbers of heat paths (N) from the three, multiply the value of b_{haz} by $3/N$.

Table 1.01 HAZ softening factor (ρ_{haz})

For all alloys supplied as extrusions, sheet, plate, drawn tubes and forging in the O and F condition, ρ_{haz} = 1,0.			
Extrusions, sheet, plate, drawn tube and forging in the T4, T5 and T6 condition			
Alloy Series	Condition	ρ_{haz} (MIG)	ρ_{haz} (TIG)
6xxx	T4	1,0	--
	T5	0,65	0,60
	T6	0,65	0,50
7xxx	T6	0,80 [a)] 1,0 [b)]	0,60 [a)] 0,80 [b)]
Sheet, plate or forging in the work hardened (H) condition			
Alloy Series	Condition	ρ_{haz} (MIG)	ρ_{haz} (TIG)
5xxx	H22	0,86	0,86
	H24	0,80	0,80
3xxx	H14, 16, 18	0,60	0,60
1xxx	H14	0,60	0,60

a) apply when a tensile stress acts transversely to the axis on a butt or a fillet weld;
b) apply for all other conditions, i.e. a longitudinal stress, a transverse compressive stress or a shear stress.



TALAT Training in Aluminium Application Technologies | **Width of heat affected zone (b_{haz})** | 2301.01.09

1.08 Stress-strain relationship

One of the first difficulties, when dealing with aluminium alloys is the problem with defining its stress-strain relationship. Even materials of the same alloys can have different stress-strain relationships. This is because of the manufacturing and heating processes the material is subjected to.

The elastic limit, often defined as the $f_{0,2}$ - limit for aluminium, is not enough for defining the stress-strain relationship for the material. It is also necessary to include the variations in Young's modulus and the strain hardening of the material. These factors are the reason why the stress-strain curve is different for each alloy.

These factors are also the main reasons why analysis of structural elements cannot be based upon simplified stress-strain relationships as for steel. Analysis must be based upon generalized inelastic stress-strain relationships. The most commonly used is the Ramberg-Osgood law, shortly presented in the following.

The Ramberg-Osgood law

A generalized law $\varepsilon = \varepsilon(\sigma)$ has been proposed by Ramberg and Osgood for aluminium alloys as

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{B}\right)^n \tag{1.03}$$

where E is the Young's modulus at the origin. Parameters B and n have to be determined by experiment. Often $B = f_{0,2} / 0{,}002^{1/n}$ is used. Then:

$$\varepsilon = \frac{\sigma}{E} + 0{,}002\left(\frac{\sigma}{f_{0,2}}\right)^n \tag{1.05}$$

Aluminium has been classified according to n as follows:

n < 10 - 20	non heat-treated alloys
n > 20 - 40	heat-treated alloys

Further descriptions of the Ramberg-Osgood law and other stress-strain relationships are given in Eurocode 9, Annex E.

2 Design basis

2.01 Basic values of strength

Characteristic values (from Eurocode 9) for the strength of wrought aluminium alloys, f_o for the 0,2 proof strength, and f_a for the ultimate strength are given in table 2.01 for some often used aluminium alloys. The characteristic strength of material influenced by welding, $f_{haz} = \rho_{haz} f_a$, is also given in the table. MIG welding is assumed.

The values of f_o and f_a are minimum values for each alloy and temper within the thickness limits given in the table. For instance, the strength f_o = 250 N/mm^2 for EN-AW 6082 given in the table is actually valid for extruded profiles with $t \leq 5$ mm. For 5 mm < $t \leq 25$ mm, f_o = 260 N/mm^2, which also apply for plates with 0,4 mm < $t \leq 6$ mm.

The characteristic values apply for structures with an operation temperature lower than 100°C. At elevated temperatures the values are reduced. For structures subject to elevated temperatures associated with fire, see EN 1999-1-2. At temperatures below 0°C the strength and elongation at rupture are often somewhat *larger*.

A characteristic value corresponds to, or is presumed to correspond to, a certain fractile for the statistical distribution of the actual parameter. The 5%-fractile is normally used as strength parameter. For metals, values in standards are usually corresponding to the 1%-fractile. Only one value out of one hundred is then allowed to be lower than the characteristic value.

2.02 Design values of strength

Design values of strength at the ultimate limit state *may be defined* as follows:

For the 0,2 proof strength $f_{o,d}$

$$f_{o,d} = \frac{f_o}{\gamma_{M1}} \tag{2.01}$$

For the ultimate strength $f_{a,d}$

$$f_{a,d} = \frac{f_a}{\gamma_{M2}} \tag{2.02}$$

$f_{o,d}$ and $f_{a,d}$ refer to tensile stresses and compression stresses as well. Values for often used alloys are given in table 2.02. The partial safety factors γ_{M1} and γ_{M2} are given in 0.04

Table 2.01 Minimum characteristic values of yield strength f_o, ultimate strength f_a and strength f_{haz} in the heat-affected zone for some wrought aluminium alloys

Alloy EN-AW	Temper	Sheet/ Extrusion	Thickness		F_o	f_a	f_{haz}	A_{50}	
			over	up to	N/mm²	N/mm²	N/mm²	%	
5083	O/H111	Sheet	2	80	115	270	270	14	
	F/H112	Extrusion		200	110	270	270	12	
	H24/H34	Sheet	2	25	250	340	272	4	
	H24/H34	Extrusion		5	235	300	240	4	
6005A	T6	Extrusion		25	200	250	163	8	
6063	T6	Extrusion		20	170	215	140	8	
6082	T4	Sheet	4	12	110	205	205	12	
		Extrusion		25	110	205	205	14	
	T6	Sheet	4	125	255	300	195	6	
		Extrusion	5	150	250	290	190	8	

$f_{haz} = \rho_{haz} \; f_a$ (MIG welding)
Sheet = Sheet, strip and plate
Extrusion = Extruded profile, extruded tube and extruded rod (not drawn tube)

Table 2.02 Design values of yield strength $f_{o,d}$, ultimate strength $f_{a,d}$ and strength $f_{haz,d}$ in the heat-affected zone for some wrought aluminium alloys

Alloy EN-AW	Temper	Sheet/ Extrusion	Thickness		$f_{o,d}$ N/mm²	$F_{a,d}$ N/mm²	$f_{haz,d}$ N/mm²	a)	b)
			over	up to					
5083	O/H111	Sheet	2	80	105	216	216	2,07	2,07
	F/H112	Extrusion		200	100	216	216	2,16	2,16
	H24/H34	Sheet	2	25	227	272	218	1,20	0,96
	H24/H34	Extrusion		5	214	240	192	1,12	0,90
6005A	T6	Extrusion		25	182	200	130	1,10	0,72
6063	T6	Extrusion		20	155	172	112	1,11	0,72
6082	T4	Sheet	4	12	100	164	164	1,64	1,64
		Extrusion		25	100	164	164	1,64	1,64
	T6	Sheet	4	125	232	240	156	1,03	0,67
		Extrusion	5	150	227	232	150	1,02	0,66

$f_{o,d} = f_o / \gamma_{M1}$ $\quad f_{a,d} = f_a / \gamma_{M2}$ $\quad f_{haz,d} = \rho_{haz} f_a / \gamma_{M2}$

a) $\mathbf{f_{a,d} / f_{o,d} = (f_a / \gamma_{M2}) / (f_o / \gamma_{M1})}$

b) $fhaz / f_{o,d} = (\rho_{haz} f_a / \gamma_{M2}) / (f_o / \gamma_{M1})$ (MIG welding)

2.03 Design values for reduced strength in the heat-affected zone

A design value for the material in the heat-affected zone *may be defined* by

$$f_{haz,d} = \rho_{haz} \frac{f_a}{\gamma_{M2}} \qquad (2.03)$$

The design value of strength in the heat affected zone is given in table 2.02.

2.04 Partial coefficients (Resistance factors)

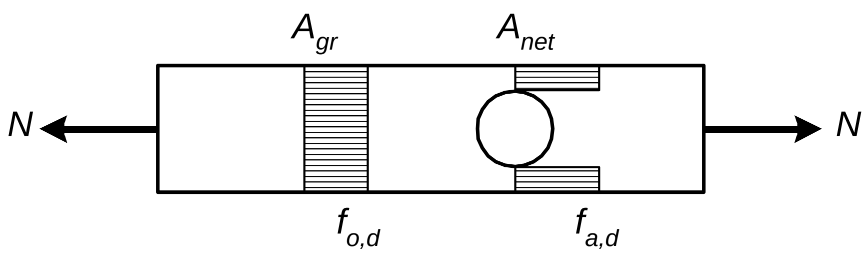
The partial safety factor for *bending and overall yielding* in tension and compression is γ_{M1} for all cross section classes. It refers to the yield strength f_o and the effective cross section allowing for local buckling and HAZ softening but with no allowance for holes. The value of γ_{M1} is

$$\gamma_{M1} = 1{,}1 \qquad (2.04)$$

The partial safety factor γ_{M2} is used for *the local capacity in net section* in tension or compression. It refers to the ultimate strength f_a and the net cross section with allowance for holes and HAZ softening but no allowance for local buckling. The value of γ_{M2} is

$$\gamma_{M2} = 1{,}25 \qquad (2.05)$$

The design expressions for the resistance of a tension member are summarized in **Figure 2301.02.01**. For further information of tensile force resistance, see 5.02



$$N_{Rd} = \min \begin{pmatrix} f_{o,d} A_{gr} \\ f_{a,d} A_{net} \end{pmatrix} \qquad f_{o,d} = f_o/\gamma_{M1} \qquad f_{a,d} = f_a/\gamma_{M2}$$

TALAT Training in Aluminium Application Technologies | Tension resistance | 2301.02.01

2.05 Gross section / net section

Fastener holes in the tension flange need not be allowed for provided that for the tension flange:

$$0{,}9\,\frac{A_{net}}{A_{gr}} \geq \frac{f_o/\gamma_{M1}}{f_a/\gamma_{M2}} \qquad (2.06)$$

When A_{net}/A_{gr} is less than this limit, a reduced flange area may be assumed.

(f_a/γ_{M2}) / (f_o/γ_{M1}) is given in table 2.02 column a). It shows that practically no holes can be made in EN-AW 5083/H24/H34 or 6xxx/T6 members without reduction in design resistance. On the other hand, in members in O, F and T4 temper, large holes can be made ($A_{net}/A_{gr} \leq 0{,}5$ in O and F temper and $\leq 0{,}35$ in T4 temper).

Furthermore, in table 2.02 column b), (f_{haz}/γ_{M2}) / (f_o/γ_{M1}) is given. It shows that, in H24/H34 and T6 material, a cross weld will always reduce the design resistance of a member in tension. In O, F and T4 temper material, however, a cross weld in a tension member does not reduce the design resistance.

3 Local buckling

3.01 Cross section classes

The resistance of a cross-section part in compression is generally limited by local buckling. The buckling load depends on the slenderness of the cross-section part. The slenderness ratio of the cross section part is normally determined by the ratio of the width divided by the thickness ($\beta = b/t$). In many cases the more general parameter for slenderness, λ, is used

$$\lambda = \sqrt{\frac{f_o}{f_{cr}}} \tag{3.01}$$

where f_o is the 0,2-limit and f_{cr} is the elastic buckling stress for a perfect plate without initial buckles or residual stresses. λ is proportional to b/t and $\sqrt{f_o/E}$ and depends on the loading and the boundary conditions, e.g. the connection to other cross sectional elements. Examples of the slenderness ratios are given in 4.03.

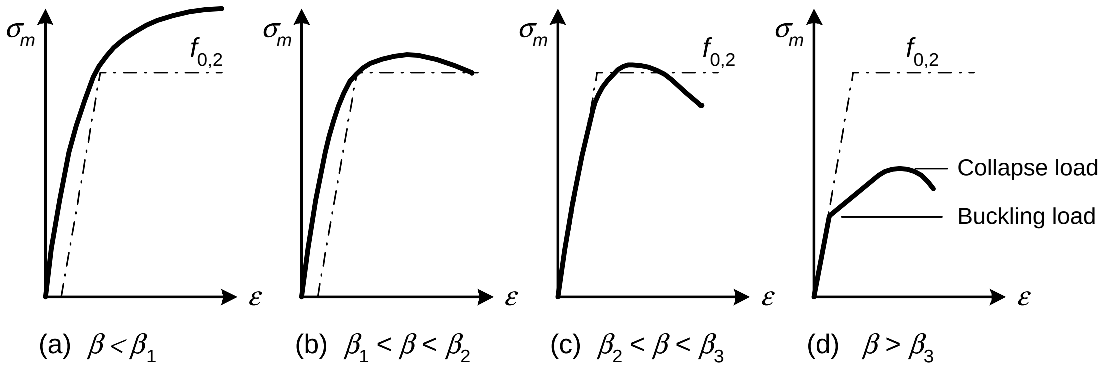
The behaviour of an element in compression depends on the slenderness ratio.

a. If the slenderness ratio of the element is small ($\beta < \beta_1$) no buckling occurs. The average stress is equal to or even larger than the ultimate strength of the material in tension, cf. **Figure 2301.03.01a**.

b. If the slenderness ratio is somewhat larger ($\beta_1 < \beta < \beta_2$) buckling occurs after the compressed element has been plastically deformed to a strain, which is more than about twice the strain corresponding to the $f_{0,2}$ (ε . 1%).

c. If the slenderness ratio is further increased ($\beta_2 < \beta < \beta_3$), buckling occurs once the 0,2 proof strength has been reached and plastic deformation has started. See **Figure 2301.03.01c**.

d. If the slenderness ratio is large ($\beta > \beta_3$), then buckling occurs before the average stress in the compressed part of the section has reached the 0,2 proof strength.



TALAT	Principle relationship between mean stress σ_m and compression ε for different slenderness β	2301.03.01

Failure normally does not occur when some cross sectional element starts to buckle, but after redistribution of stresses and yielding.

The division of cross-sections into four classes for *members in bending* corresponds to the different behaviour as above. Class 1 and 2 cross sections have compact cross-section parts that behave according to *a* and *b*. Class 3 cross sections have semi-slender cross section parts and behave according to *c*. Class 4 cross sections have one or more slender section parts that behave according to *d*. See 4.01 - 4.04.

For a *member in axial compression*, actually only two classes are of interest: *non-slender sections* with class 1 - 3 cross section parts and *slender sections* with one or more slender section parts that behave according to *d*. See 5.033.

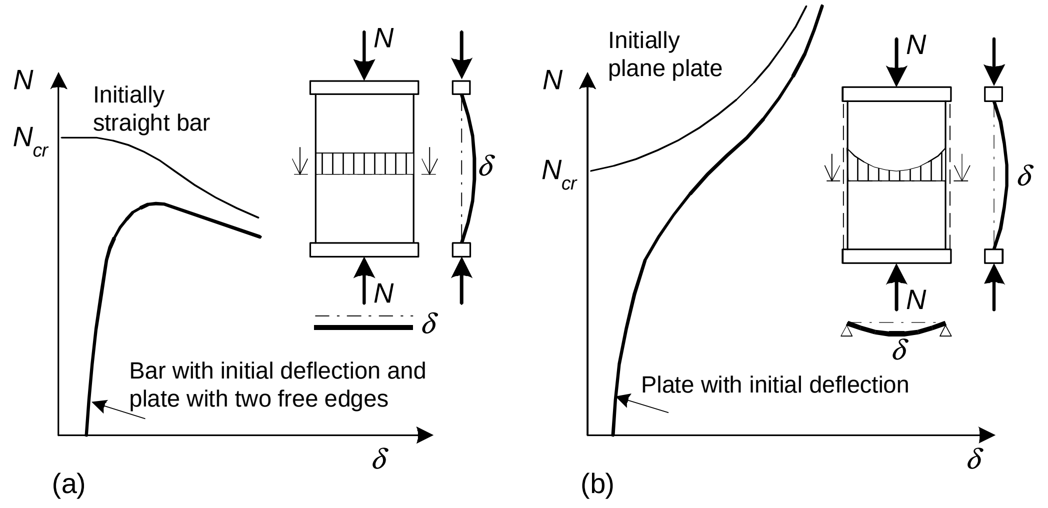
3.02 Behaviour of slender plates

A slender plate with four edges, all simply supported, can carry an ultimate load that is greater than the critical buckling load according to the theory of elasticity for perfectly flat plates. Figures 3.02a and 3.02b show the difference between a plate with two free edges (**Figure 2301.03.02 a**) and a plate supported along four edges, (**Figure 2301.03.02 b**).

A plate with two longitudinally unsupported edges will buckle in the same way as a compressed bar. Hereby, the plate will deform to a surface of single curvature (disregarding small disturbances along the unsupported edges). Every vertical strip will have the same strain and deflection.

After buckling, a plate with four edges, all simply supported, will deform to a surface of double curvature. When compression increases, the strip in the centre will behave in a different way compared to one along the edge. The strip along the edge will remain straight and the compression will result in increased stresses. The central strip, on the contrary, will deflect without any particular increase in stress.

This description of the behaviour of a rectangular plate with simply supported edges, applies only under the condition that the compressed edges are still straight after the plate has buckled. In a long plate that buckles into a number of half-waves, the node lines act as straight loaded edges.



(a) compressed bar and plate with two free edges and
(b) plate supported along all four edges

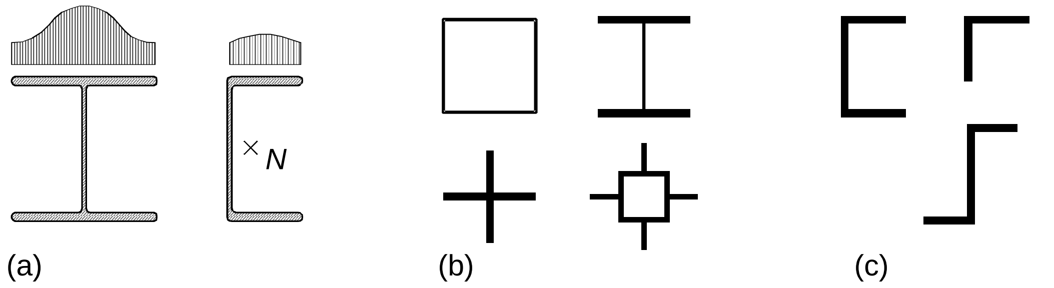
TALAT	Principle relationship between maximum deflection δ and load *N*, with and without initial deflections	2301.03.02

The behaviour is also similar for a rectangular plate with three supported edges and one free edge (an outstand element), where the last edge is parallel to the direction of the load. The same condition applies here as in the previous case, if the loaded edges stay straight after buckling and if they are parallel and in the same plane. The compression flange of an I-beam is an example of this case. Each half flange acts as a plate supported along three edges and, after buckling into a number of half-waves, the node lines correspond to the straight, loaded edges.

When the edge of the flange buckles, redistribution of stresses occur with an increase of stresses close to the web and a decrease of stresses at the edges of the flange, see **Figure 2301.03.03a**.

For a single sided flange, as in a channel section, the behaviour is usually different. When the free edge of the flange buckle, the ends of the beam rotate and a redistribution of stresses from the free edge to the connected edge does not occur if the load still act in the centre of the cross section (pinned ends). Therefore, the buckling stress usually is an upper limit for the resistance of an unsymmetrical cross section with outstands.

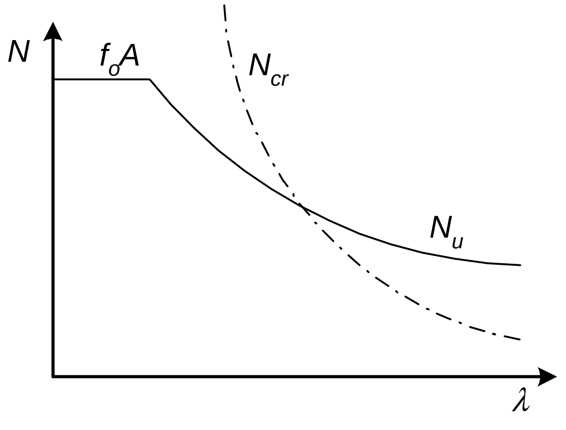
For sections composed entirely of radiating outstands, such as angles, tees and cruciforms, local and torsional buckling are closely related. Usually, only torsional buckling (lateral-torsional for tees) need to be checked.



(a) Stress distribution after buckling of a flange of an I-section and a flange of a channel section with pinned ends
(b) Cross sections with stable post-buckling behaviour
(c) Cross section with no post-buckling strength

TALAT	Relation between stress distribution and flange buckling	2301.03.03

The load exceeding the buckling load is usually called post buckling range (post buckling strength, post critical range). A stable post buckling range exists for an internal cross section element for most load cases such as axial load, bending moment and transverse load. For cross section parts with one free edge, a stable post buckling range exists in flanges of I-profiles and composite cross sections as in **Figure 2301.03.03b**, but normally not in flanges of channels, angles and Z-sections as in **Figure 2301.03.03c**.



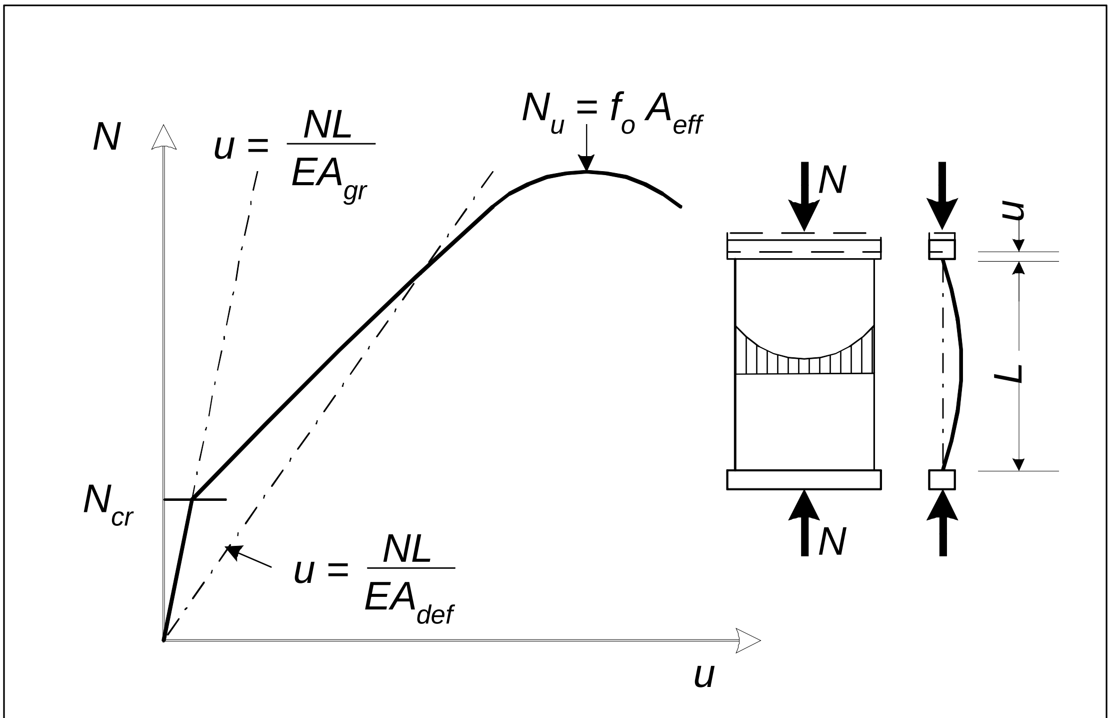
TALAT	Principle relationship between slenderness parameter λ, resistance N_u and buckling load $\boldsymbol{N_{cr}}$	2301.03.04

The resistance in the post buckling range is of importance for slender plates. The post buckling strength increases with increasing slenderness ratio. See **Figure 2301.03.04**.

3.03 Effective cross section

The relation between compression *u* and axial load *N* for a slender plate is illustrated in

Figure 2301.03.06. This is characterized by the use of the term *effective area*. The effective area, as in many regulations for cold formed structures, can be determined by multiplying the *effective width* with the gross thickness of the plate, or, as in Eurocode 9, by multiplying an *effective thickness* with the *gross width*.



TALAT Training in Aluminium Application Technologies	Principle relationship between compression *u* and axial load *N*	2301.03.05

A_{gr}: area of gross cross section, used for calculation of deformation prior to buckling

A_{def}: area of effective cross section used for calculation of deformation. A_{def} depends on the load level

A_{eff}: area of effective cross section used for calculation of resistance.

In many new national standards the effective thickness is used since it leads to simple calculations. In Eurocode 9, the effective thickness is used for calculation of resistance. In many cases an effective width concept is used for calculation of stiffness. See 5.033 and 6.03.

Effective cross section for calculation of resistance is defined as

$$N_u = A_{eff}\ f_o \qquad (3.02)$$

where N_u is the rupture load for the cross section part.

The axial deformations can be calculated from the relationship

$$u = \frac{N\,L}{E\,A_{def}} \qquad (3.03)$$

where A_{def} depends on the load level.

The definition of A_{def} is different from that for A_{eff}, and, in principle, $A_{def} \approx A_{eff}$ even just before failure.

3.04 Calculation technique for class 4 cross sections

The resistance of many structures is independent of the overall bending deflection, as in the case of a short, centric compressed column. The ultimate load is governed only by local buckling, i.e. of A_{ef}. For a beam in bending, the resistance is determined by the section modulus W_{eff} for the effective cross section.

The resistance for other structures may also depend on the bending stiffness. One example is the resistance of a long column which depends on the bending stiffness as well as the strength of the most compressed cross section part. The bending stiffness is determined by the deformation of the compressed cross section part expressed by I_{def}. In 5.033 a method is given for determination of the resistance for a compressed column with an arbitrary cross section. The method is based on the effective area with regards to strength, A_{eff}, and the bending stiffness I_{def} based on the effective area for deflections, A_{def}. The method is also used for a compressed stiffener according to 6.03.

3.05 Calculation of deflections of beams with class 4 cross section

To calculate the deflection of a beam with class 4 cross section is very complicated, due to the fact that the effective stiffness varies with the load level and along the beam. In Eurocode 9, a simplified procedure is used, which mean that only the second moment of area I_{eff} for the effective cross section in the ultimate limit state need to be calculated besides I_{gr} for the gross cross section.

The calculation of elastic deflection should generally be based on the properties of the gross cross section of the member. However, for slender sections it may be necessary to take reduced section properties to allow for local buckling.

Advantage may be taken from reduced stress levels for class 4 section to calculate the effective thickness, using the following fictitious second moment of area I_{fic}, constant along the beam

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{f_o}(I_{gr} - I_{eff}) \qquad (3.04)$$

where:

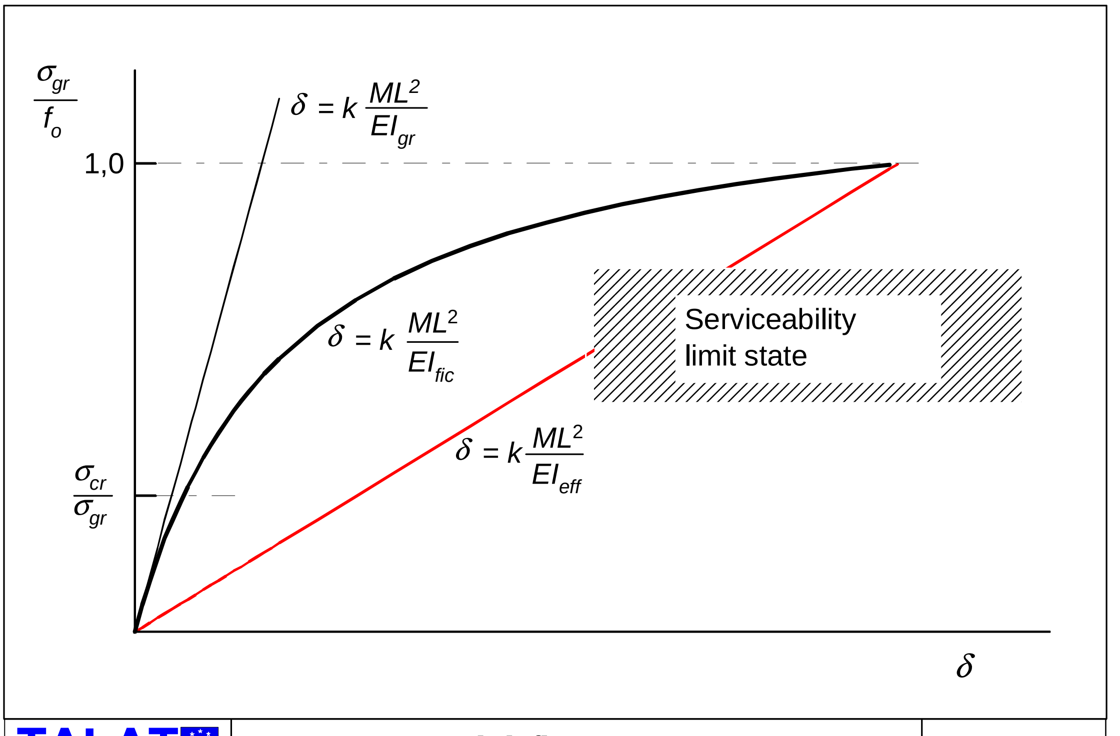
I_{gr} = second moment of area of the gross cross section

I_{eff} = second moment of area of the effective cross section in the ultimate limit state, with allowance for local buckling

σ_{gr} = maximum compressive bending stress in the serviceability limit state, based on the gross cross section (positive in the formula).

f_o = characteristic strength for bending and overall yielding

In **Figure 2301.03.06** the deflection δ is given as a function of the load level σ_{gr}/f_o. δ based on I_{gr}, I_{eff} and I_{fic} are compared. The curve based on I_{fic} is similar to curves from tests. Note that deflections are calculated at the serviceability limit state.



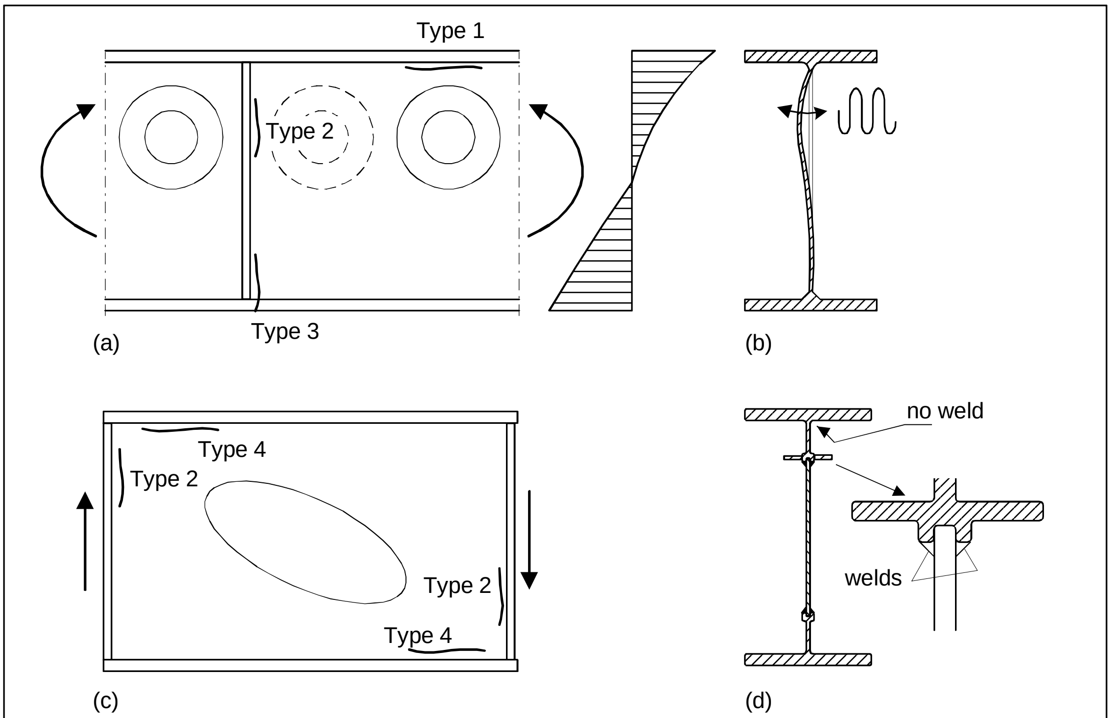
TALAT	Load-deflection curves	2301.03.06

3.06 Breathing

When a beam with a slender web is loaded in compression or bending, because of initial deflection or buckling, the web will have out-of-plane deformations. After the beam is unloaded, the web will return to the original place. If the load is cyclic, this phenomenon will repeat, which is called Abreathing≅. Because of the fluctuation of the web, plate bending stresses are created. The possibility of initiating fatigue cracks at the flange-to-web junction and along the stiffeners is increased.

The stresses in the web are not only dependent on the load, but also on the slenderness of the web. This makes the fatigue problem of web breathing complicated. Plate bending stresses normal to panel boundaries are shown to be the primary cause of fatigue cracks.

For girders in bending fatigue assessment for type 1 crack is automatically accomplished by the fatigue assessment for type 2 cracks. Type 1 cracks occur at a toe on the web side of the fillet weld to connect the web to the compression flange and type 2 cracks are observed at a toe on the web side of the fillet weld connecting the vertical stiffener to the web. See **Figure 2301.03.07a**. Breathing does not influence type 3 cracks, initiated at the fillet weld to connect the web to the tension flange, as the web does not deflect on the tension side of the web.



TALAT Training in Aluminium Application Technologies	Breathing (a) Beam in bending (b) Web breathing (c) Web in shear (d) Web with stiffener	2301.03.07

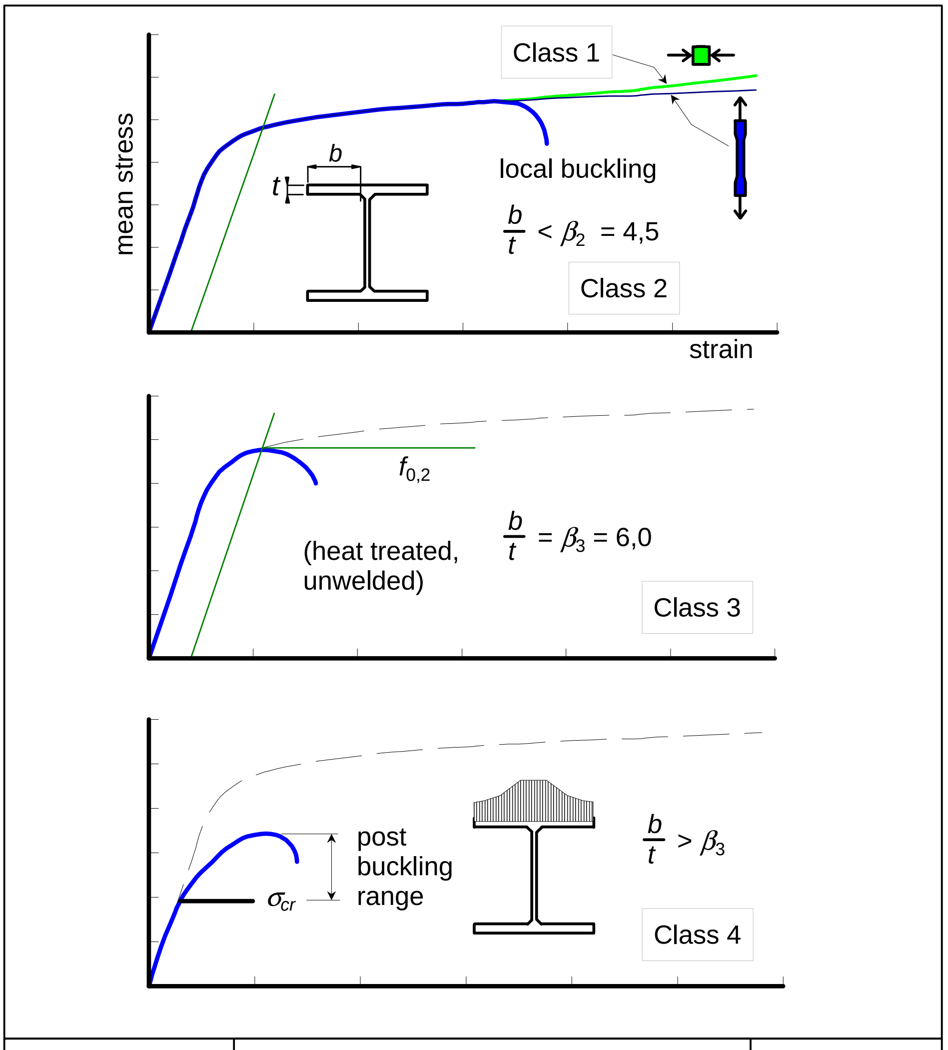
For girders in shear the bending stresses due to breathing acts together with in-plane shear stresses and tensile membrane stresses due to tension field action in the post buckling stage of a slender web. To avoid type 4 cracks (see **Figure 2301.03.07c**) due to breathing of a web in shear, it is usually recommended that the maximum applied in-plane shearing stress (at the ultimate limit state for fatigue) should be below the shear buckling stress for the simply supported condition along the four edges.

The above findings and recommendations are based on tests on steel plate girders. No tests have been found on aluminium plate girders. The fatigue strength of thin-walled aluminium plate girders can be increased substantially by the use of extruded T-shaped flanges, where there are no welds at the junction between the flange and the web. See **Figure 2301.03.07d**. Furthermore, adding stiffeners to the web of the flange profile increase the elastic buckling stresses in the web.

4 Bending moment

4.01 Yielding and local buckling

The ultimate limit state of a beam can occur in different circumstances depending upon the geometry of the beam (the span *L*, the *b/t* ratio of the individual parts etc.), the loading and support conditions and the type of connection. Failure is most often accompanied by local buckling of compressed cross section part. Exceptions are compact cross sections, such as solid rectangular and circular sections and beams made by material with small ductility.



TALAT Training in Aluminium Application Technologies	Stress-strain curves for compression flange in different cross section class	2301.04.01

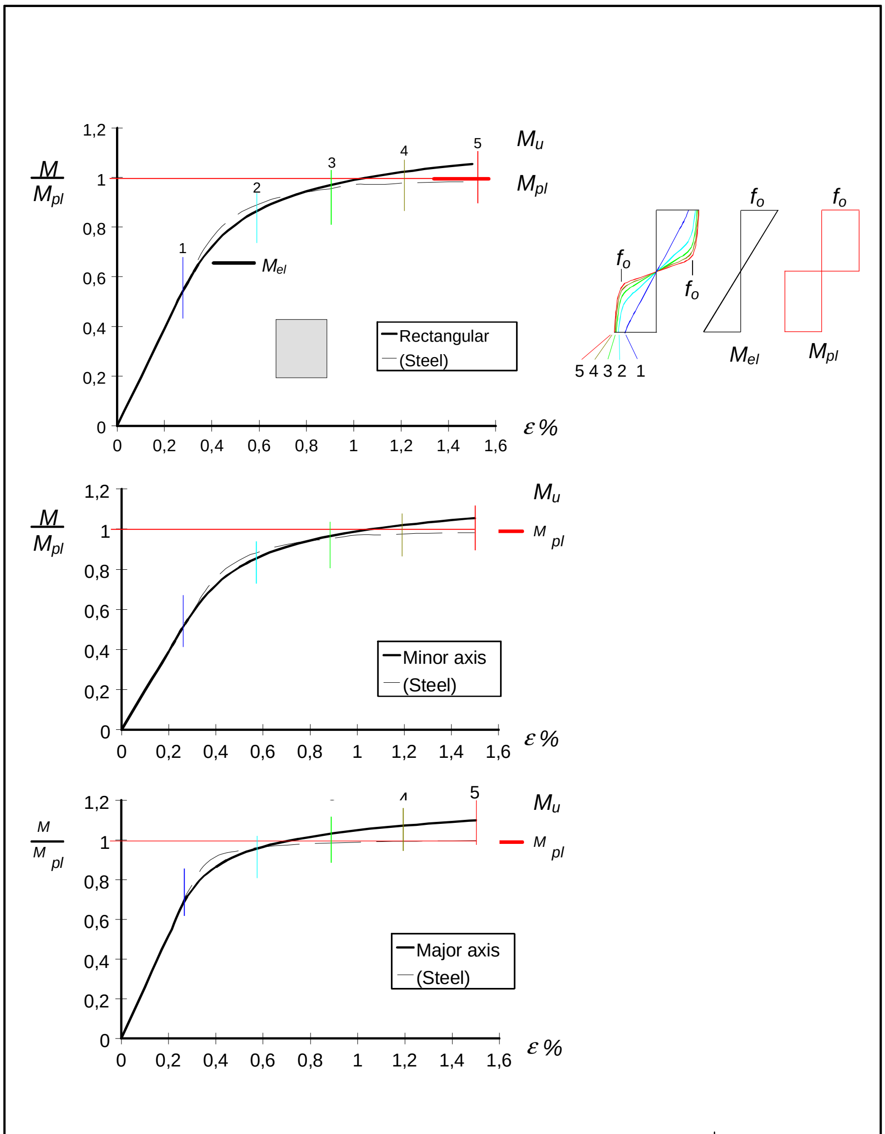
Class 1 cross section

In the case of a beam with compact cross-section, in which local buckling or flexural-torsional buckling are not likely to occur, the beam experiences the inelastic range after reaching the limiting elastic moment $M_{el} = W_{el} f_o$ until the ultimate moment M_u is reached.

This moment cannot be defined (as it is for steel structures) as the full plastic moment M_{pl} $=W_{pl} f_o$. In fact, due to the hardening behaviour of the σ–ε law of aluminium, a limiting curvature has to be defined corresponding to the limit of strain in the outermost fibre of the cross section. The increase in strength, M_u - M_{el}, obtained in this phase can be quantified through a relation

$$M_u = \alpha \; M_{el} \qquad (4:01)$$

which defines in a general form a shape factor $\alpha = \alpha_1$ which is not solely dependent upon the cross-sectional geometry, as is usual, but also depends upon the parameters in the σ–ε law and on the definition of the limiting curvature. **Figure 2301.04.02** show that $M_u > M_{pl}$ if ε is larger than about 1%. Then $\alpha_1 > M_{pl}/M_{el}$.



TALAT Training in Aluminium Application Technologies | Moment-strain curves | 2301.04.02

In this case the rotational capacity of the cross-section, which characterise the flexural ductility of the beam, allows redistribution of the internal forces, and it is therefore possible to carry out a limit analysis of the whole structure.

Class 2 cross-section

In the case of open profiles, local buckling phenomena are most likely to occur in the compressed region of the cross-section and cause a decrease in the M–ε curve of the beam. This unstable behaviour is dependent upon the $\beta = b/t$ ratio. If the decreasing portion of the curve occurs after the ultimate moment M_u is reached, the beam keeps the same maximum load-carrying capacity, but the rotational capacity of the cross-section is reduced why redistribution of the internal forces is limited. Note that using M_{pl} as the ultimate resistance corresponds to a strain at the extreme fibres of about 1% for rectangular cross sections as well as for mayor and minor axis bending of H cross-sections. See **Figure 2301.04.02**. Also note that the difference between the curves for aluminium with its hardening behaviour of the σ–ε law and steel is not that large.

Class 3 and 4 cross-section

If the decreasing portion of the curve occurs before the ultimate moment M_u is reached (class 3 cross-section, see **Figure 2301.04.01**), or even before the elastic moment M_{el} (class 4 cross-section, see **Figure 2301.04.01**), the load-carrying capacity of the beam is affected by local buckling phenomena to a higher degree if the *b/t* ratio is large (e.g. thin-walled profiles). Also ductility decreases to the extent that redistribution of internal forces cannot be considered.

4.02 Classification of cross sections

Based on the above, four classes of cross sections are defined, as follows:

- Class 1 cross sections are those which can form a plastic hinge with the rotation capacity required for plastic analysis.

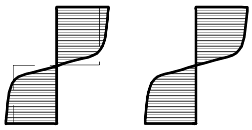
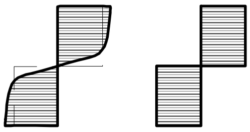
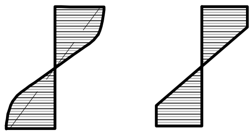
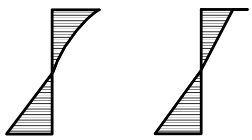
- Class 2 cross sections are those which can develop their plastic moment resistance, but have limited rotation capacity.

- Class 3 cross sections are those in which the calculated stress in the extreme compression fibre of the member can reach its proof strength, but local buckling is liable to prevent development of the full plastic moment resistance.

- Class 4 cross sections are those in which it is necessary to make explicit allowances for the effects of local buckling when determining their moment resistance or compression resistance.

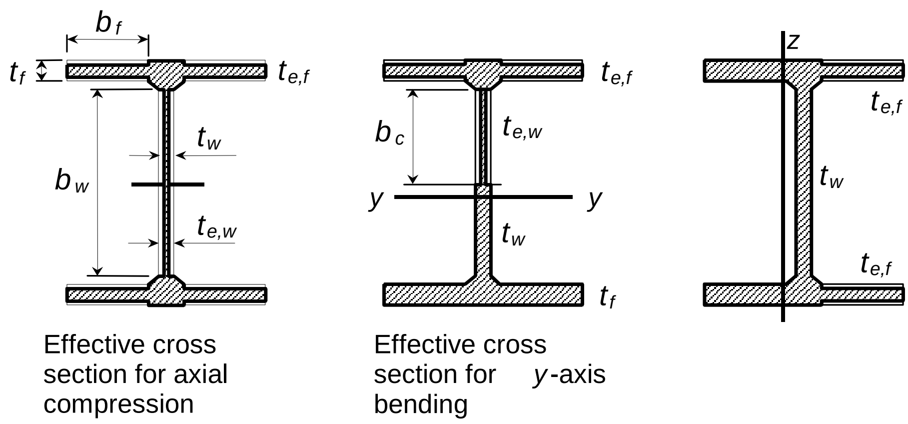
Stress distribution, shape factor and slenderness limits for the four cross section classes are summarised in table 4.01 for unwelded sections. For welded sections the same formulae are applicable, except that W_{pl}, W_{el} and W_{eff} are reduced to allow for HAZ softening. See 4.07.

Table 4.01 Stress distribution and shape factor for cross section class 1 to 4

Class	Actual and nominal stress distribution	Shape factor	Slenderness limits
1		$\alpha \geq \frac{W_{pl}}{W_{el}}$ (See Eurocode 9, Annex G)	$\beta < \beta_1$
2		$\alpha = \frac{W_{pl}}{W_{el}}$	$\beta_1 < \beta \leq \beta_2$
3		$\alpha = 1 + \frac{\beta_3 - \beta}{\beta_3 - \beta_2}\left(\frac{W_{pl}}{W_{el}} - 1\right)$	$\beta_2 < \beta \leq \beta_3$
4		$\alpha = \frac{W_{eff}}{W_{el}}$	$\beta > \beta_3$

The classification of a section depends on the proportions of each of its compression elements. The compression elements include every element of a cross section that is either totally or partially in compression.

The various compression elements in a cross section (such as a web or a flange) can, in general, be in different classes. The cross section should be classified by quoting the least favourable class of its compression elements.



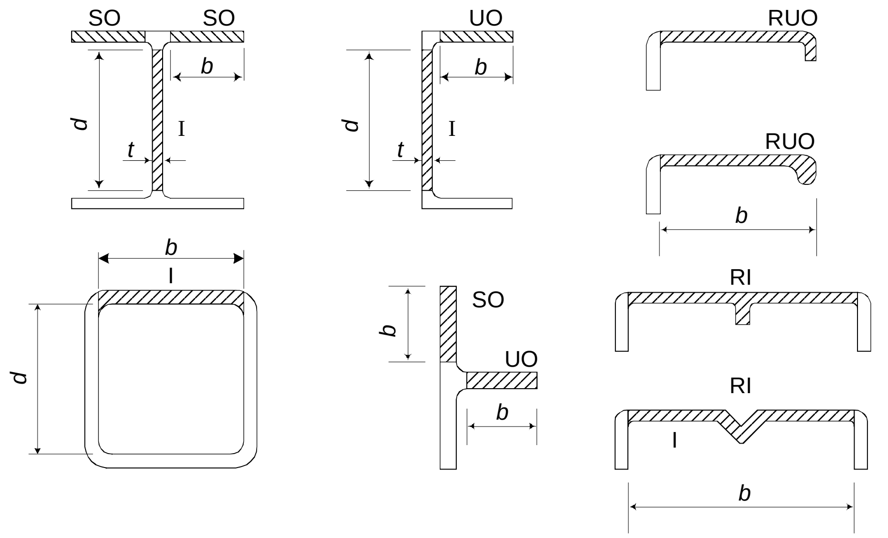
TALAT	Stress-strain curves for compression flange in different cross section class	2301.04.03

The cross section class depends on the loading. Therefore, a cross section can belong to different classes for *y*- and *z*-axis bending as well as axial force. See examples in **Figure 2301.04.03**. No cross section class or effective cross section is defined for the combined action of normal force and bending moment, but for the separate actions.

The following basic types of thin-walled elements are identified in the classification process:

a) flat outstand element
b) flat internal element
c) curved internal element.

These elements can be unreinforced, or reinforced by longitudinal stiffening ribs or edge lips or bulbs (see **Figure 2301.04.04**).



(a) ---------------- Unreinforced ---------------- (b) Reinforced

Key SO: symmetric outstand
UO: unsymmetric outstand
I: internal

RI: reinforced, internal
RUO: reinforced unsymmertical outstand

TALAT Training in Aluminium Application Technologies | Types of element | 2301.04.04

4.03 Slenderness parameter

The susceptibility of an unreinforced flat element to local buckling is defined by the slenderness parameter β, which has the following values:

a) flat outstand or internal elements with no stress gradient	$\beta = b/t$	(4.02a)
b) internal element with a stress gradient that results in a neutral axis at the centre	$\beta = 0{,}40\ b/t$	(4.02b)
c) for any other stress gradients	$\beta = g\ b/t$	(4.02c)

where b is the width of an element, t is the element thickness and g is the stress gradient coefficient. For vertical elements in **Figure 2301.04.04**, d is used instead of b in the expressions for β.

For internal elements and outstand elements with peak compression at root, g is given by the expressions:

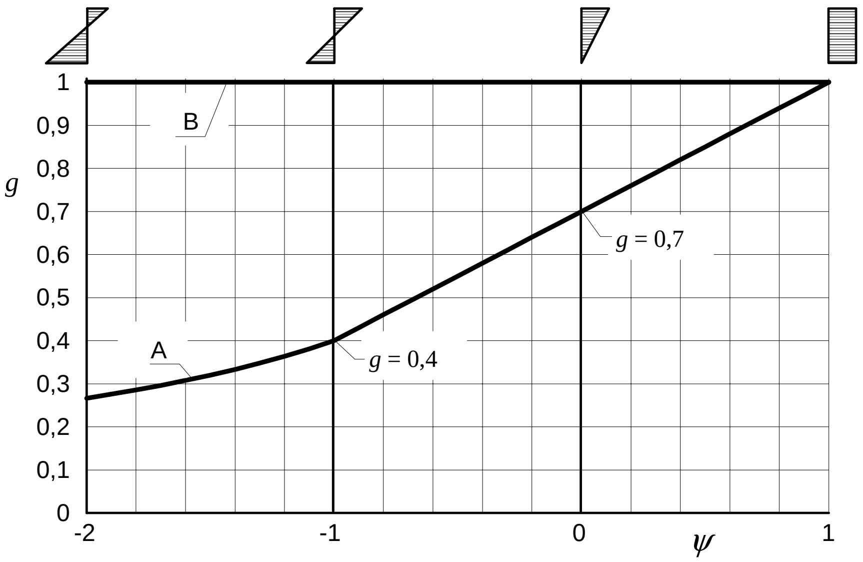
$$g = 0{,}70 + 0{,}30\,\psi \qquad (1 > \psi > -1), \qquad (4.03)$$

$$g = 0{,}80/(1 - \psi) \qquad (\psi \leq -1), \quad \text{see } \textbf{Figure 2301.04.05} \qquad (4.04)$$

where ψ is the ratio of the stresses at the edges of the plate under consideration, related to the maximum compressive stress.

For outstand elements with peak compression at toe use $g = 1$.

When considering the susceptibility of a reinforced flat element and cylindrical elements to local buckling, see Eurocode 9.



For internal elements or outstands (peak compression at root) use curve A.
For outstands (peak compression at toe) use line B

TALAT Training in Aluminium Application Technologies	Flat internal elements under stress gradient, values of g	2301.04.05

4.04 Element classification

The classification of elements in cross sections is linked to the values of the maximum slenderness parameter β. For elements in beams:

$\beta \leq \beta_1$: class 1

$\beta_1 < \beta \leq \beta_2$: class 2

$\beta_2 < \beta \leq \beta_3$: class 3

$\beta_3 < \beta$: class 4

Values of β_1, β_2 and β_3 are given in table 4.02.

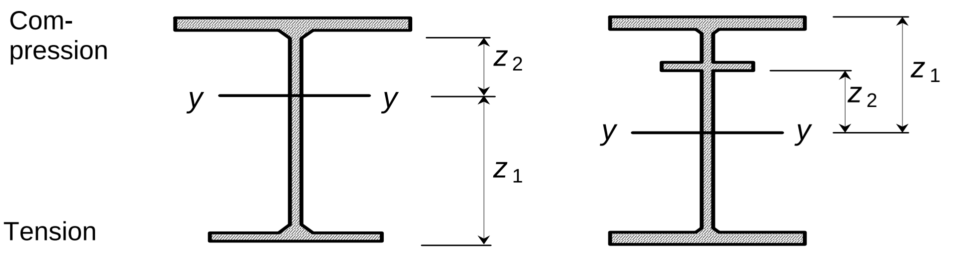
Table 4.02: Slenderness parameters β_1/ε, β_2/ε and β_3/ε

	Heat treated, unwelded			Heat treated, welded or non heat treated, unwelded			Non heat treated, welded		
Elements	β_1/ε	β_2/ε	β_3/ε	β_1/ε	β_2/ε	β_3/ε	β_1/ε	β_2/ε	β_3/ε
Outstand	3	4,5	6	2,5	4	5	2	3	4
Internal	11	16	22	9	13	18	7	11	15

$\varepsilon = \sqrt{250/f_o}$ where f_o is in N/mm^2

In the table, an element is considered welded if it contains welding at an edge or at any point within its width. However, cross sections of a member that do not contain welding may be considered as unwelded even if the member is welded elsewhere along its length. Note that in a welded element the classification is independent of the extent of the HAZ.

It is permissible to use a modified expression, $\varepsilon = \sqrt{250\, z_1/(f_o\, z_2)}$ when classifying elements in members under bending, if the elements are less highly stressed than the most severely stressed fibres in the section. In this expression, z_1 is the distance from



TALAT Training in Aluminium Application Technologies | Definition of z_1 and z_2 | 2301.04.06

the elastic neutral axis of the effective section to the most severely stressed fibres, and z_2 is the distance from the elastic neutral axis of the effective section to the element under consideration. See **Figure 2301.04.06**. In principal z_1 and z_2 should be evaluated on the effective section by means of an iterative procedure. Normally two steps are sufficient, see 4.06.

4.05 Effective thickness

Local buckling in class 4 members is generally allowed for by replacing the true section by an effective section. The effective section is obtained by employing a local buckling coefficient ρ_c to factor down the thickness. ρ_c is applied to any uniform thickness class 4 element that is wholly or partly in compression. Elements that are not uniform in thickness require special study by the designer.

The coefficient ρ_c is found separately for different elements of the section, in terms of the ratio β/ε, where β is found as in 4.03 and ε is defined under table 4.02.

Values of ρ_c are as follows:

$$\rho_c = 1{,}0 \qquad \text{when } \beta/\varepsilon \leq \beta_3/\varepsilon\,, \qquad (4.05)$$

$$\rho_c = \frac{A}{\beta/\varepsilon} - \frac{B}{(\beta/\varepsilon)^2} \qquad \text{when } \beta/\varepsilon > \beta_3/\varepsilon \qquad (4.06)$$

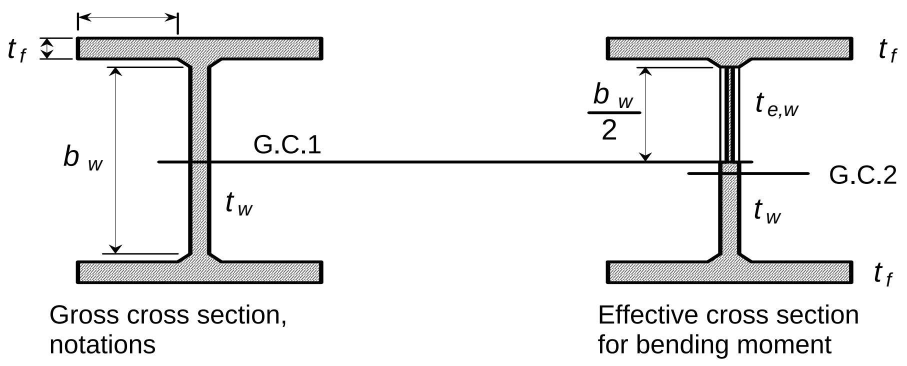
where β_3/ε, A and B are given in table 4.03.

Table 4.03 Slenderness parameter β_3/ε and coefficients A and B in the expression for ρ_c

Elements	Heat treated, unwelded			Heat treated, welded or non heat treated, un-welded			Non heat treated, welded		
	β_3/ε	A	B	β_3/ε	A	B	β_3/ε	A	B
Outstand	6	10	24	5	9	20	4	8	16
Internal	22	32	220	18	29	198	15	25	150

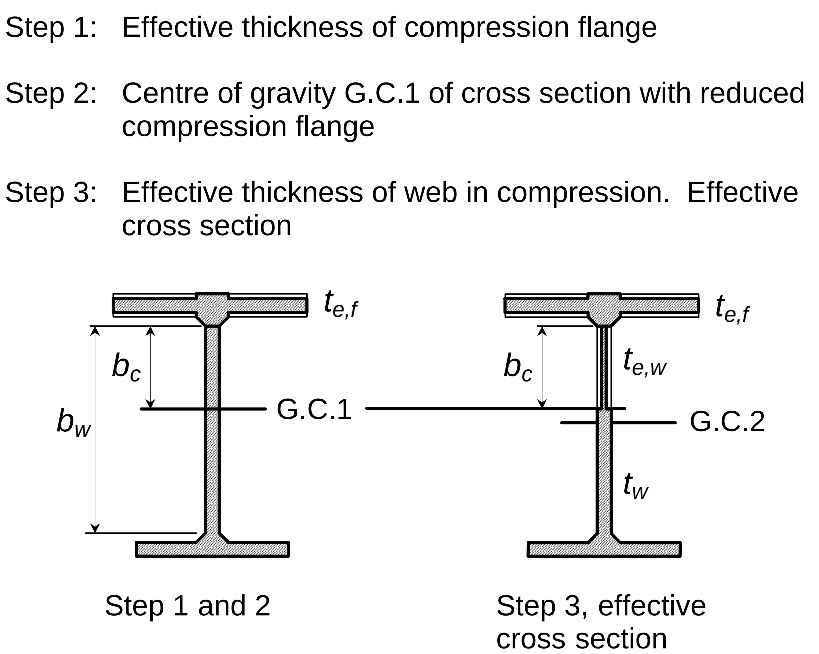
4.06 Effective cross section

The effective parts of a class 4 cross section are combined into an effective cross section. The effective cross section depends on the load case. An example of effective cross section for a symmetric I-girder with class 1, 2 or 3 flanges and class 4 web is given in **Figure 2301.04.07**. Note that no iteration process is necessary. The effective thickness of the web is based on the width b_w and ψ = -1,0. The web thickness is reduced to the effective thickness $t_{w,ef}$ within $b_w/2$ on the compression side only.



TALAT Training in Aluminium Application Technologies	**Effective cross section for an I-girder with equal class 1,2 or 3 flanges**	2301.04.07

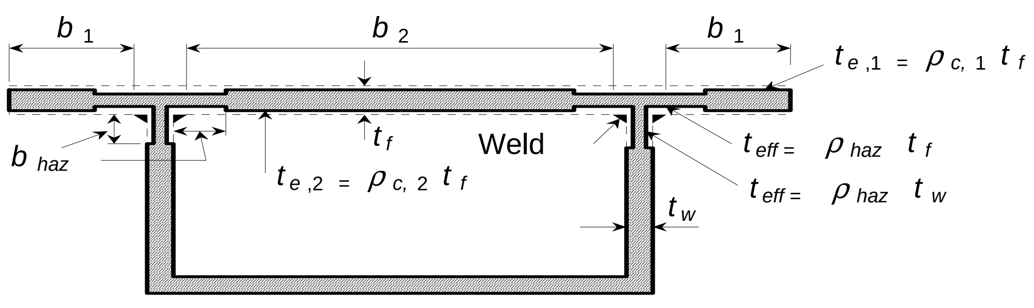
Calculation steps for a symmetric I-girder with class 4 flanges and web or an un-symmetric I-girder are given in **Figure 2301.04.08**. The effective thickness of the web is based on the width b_w and $\psi = 1 - b_w/b_c$ where b_c is the width of the compression part of the web calculated for the cross section with reduced compression flange but unreduced web (Step 2). The web thickness is reduced to the effective thickness $t_{w,ef}$ within b_c (Step 3).



TALAT Training in Aluminium Application Technologies	Effective cross section for an I-girder with different flanges	2301.04.08

4.07 Welded section

In welded sections the effective thickness is obtained using a reduced thickness $t_{eff} = \rho_{haz}\, t$ for the HAZ material or the reduced thickness $t_{eff} = \rho_c\, t$ for class 4 elements, whichever is the smaller (but $t_{eff} \leq t$). An example of effective section is shown in **Figure 2301.04.09**. $\rho_{c,1}$ is based on b_1/t_f for the outstand parts of the compression flange and $\rho_{c,2}$ is based on b_2/t_f for the internal part of the compression flange. b_{haz} and ρ_{haz} are given in 1.07. For the example in **Figure 2301.04.08** $\rho_{haz} < \rho_c$



TALAT Training in Aluminium Application Technologies	Effective section of a welded hollow section	2301.04.09

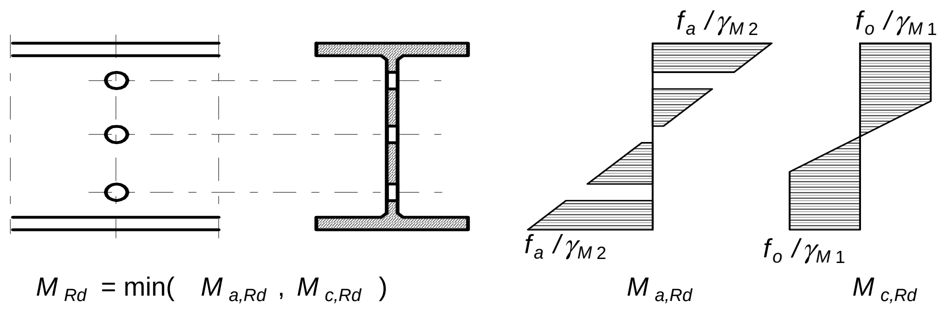
4.08 Section with holes

In a section with holes the design value of the bending moment resistance M_{Rd} is the lesser of $M_{a,Rd}$ and $M_{c,Rd}$, where $M_{a,Rd}$ is based on the design value of the ultimate strength f_u/γ_{M2} in the net section and $M_{c,Rd}$ is based on the design value of the yield strength f_o/γ_{M1} with no allowance for holes. For both $M_{a,Rd}$ and $M_{c,Rd}$, allowance should be made for HAZ-softening, if any.

$$M_{a,Rd} = W_{net} \frac{f_u}{\gamma_{M2}} \quad \text{in the net section} \qquad (4.07)$$

$$M_{o,Rd} = \alpha\, W_{el} \frac{f_o}{\gamma_{M1}} \quad \text{at each cross-section} \qquad (4.08)$$

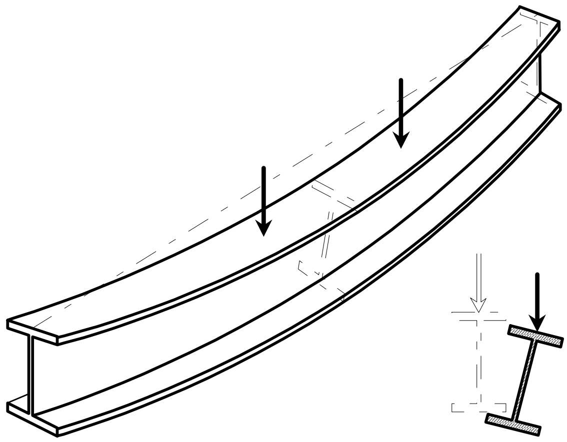
where W_{net} is the elastic modulus of the net section allowing for holes and HAZ softening, if welded. The stress distribution for $M_{a,Rd}$ and $M_{c,Rd}$ is illustrated in **Figure 2301.04.10** for a class 3 cross section beam with holes in the web.



TALAT Training in Aluminium Application Technologies	Nominal stress distribution in a section with holes	2301.04.10

4.09 Lateral torsional buckling

Lateral torsional buckling is a failure mode caused by bending moment and/or transverse load resulting in twisting and bending perpendicular to the plane of the load. The risk for lateral buckling is largest in beams with small torsional stiffness and small lateral bending stiffness. See **Figure 2301.04.11**.



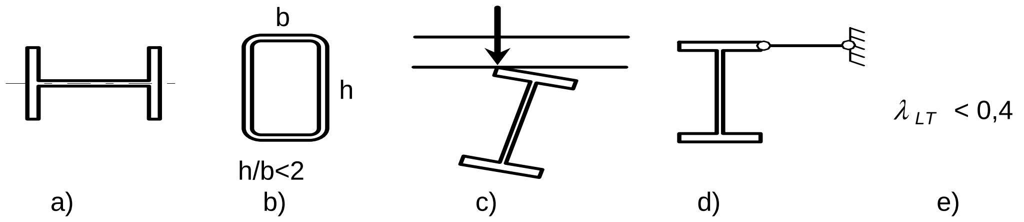
TALAT Training in Aluminium Application Technologies	Lateral-torsional buckling	2301.04.11

Lateral torsional buckling need not be checked in any of the following circumstances:

a) Bending takes place about the minor principal axis.
b) The compression flange is fully restrained against lateral movement throughout its length
c) The slenderness parameter $\overline{\lambda}_{LT}$ between points of effective lateral restraint is less

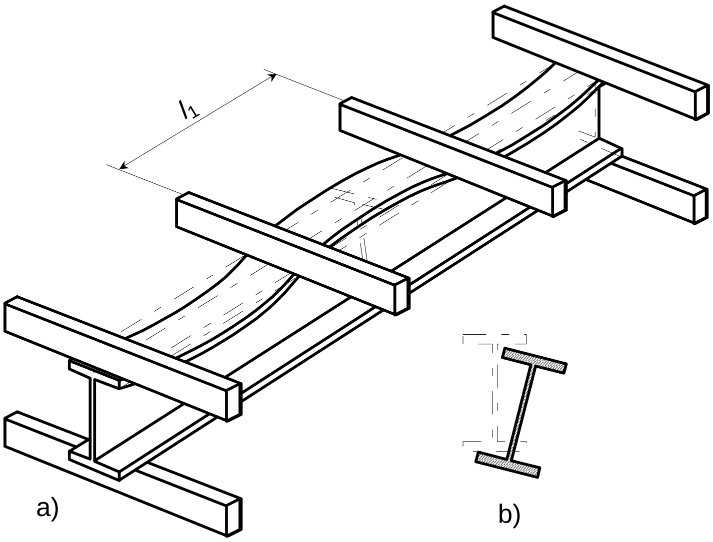
than 0,4.

The loading structure often has a bracing effect. One example is the case of a slab through which load is applied to the upper flange of an I-section. See **Figure 2301.04.12c**. When the slab is loaded, stabilizing twisting moments develop, preventing twisting and hence lateral torsional buckling.



Cases for which lateral-torsional buckling need not be checked — 2301.04.12

Lateral-torsional buckling does not occur for round sections or rectangular hollow cross sections where the height is less than twice the width.



Lateral-torsional buckling between restraints — 2301.04.13

A beam with the upper flange intermittently braced, e.g. by purlins, is a common case where lateral buckling between the bracing may occur, cf. **Figure 2301.04.13**. For this case the check d) above can be simplified to:

$$\frac{l_1}{i_f} < 42\varepsilon \qquad \text{which correspond to } \bar{\lambda}_{LT} < 0{,}4 \qquad (4.09)$$

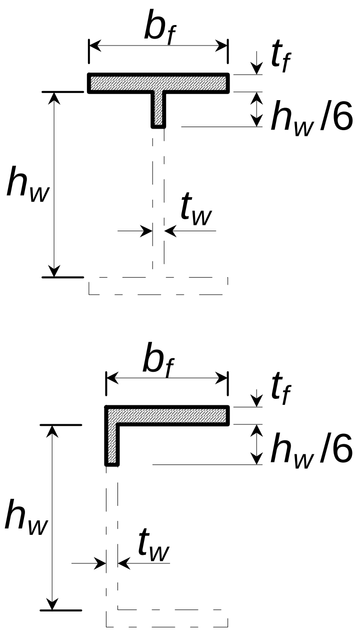
In this expression, l_1 is the distance between the bracing and i_f is the radius of inertia for a cross section consisting of the compressed flange and 1/6 of the web, see **Figure 2301.04.14**.
The expression can also be used for unsymmetrical beams, e.g. channels, if the beams are both laterally and torsionally restrained by the purlins.

In the case of continuous plate girders with the upper flange restrained, there is a kind of instability where the lower flange will deflect in the vicinity of a support. If a slab restrains the upper flange, the beam is stable if the ratio b_w/t_w for the web is less than 33.

$$i_f = \frac{b_f}{\sqrt{12}} \frac{1}{\sqrt{k}} \qquad \text{(I-section)} \qquad (4.10)$$

$$i_f = \frac{b_f}{\sqrt{12}} \sqrt{\frac{4}{k} - \frac{3}{k^2}} \qquad \text{(Channel)} \qquad (4.11)$$

$$\text{where } k = 1 + \frac{h_w t_w}{6 b_f t_f} \qquad (4.12)$$



TALAT Training in Aluminium Application Technologies | Lateral torsional buckling between bracing | 2301.04.14

Reduction factor for lateral torsional buckling

The reduction factor χ_{LT} depends upon the cross section class and the slenderness parameter $\bar{\lambda}_{LT}$. χ_{LT} is taken from the diagram in **Figure 2301.05.15** or is determined from:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \tag{4.13}$$

$$\phi_{LT} = 0,5 \left[1 + \alpha_{LT} \, (\lambda - \bar{\lambda}_{0,LT}) + \bar{\lambda}_{LT}^2 \right] \tag{4.14}$$

The value of α_{LT} and $\bar{\lambda}_{0,LT}$ should be taken as:

α_{LT} = 0,10 and $\bar{\lambda}_{0,LT}$ = 0,6 for class 1 and 2 cross sections and (4.15)

α_{LT} = 0,20 and $\bar{\lambda}_{0,LT}$ = 0,4 for class 3 and 4 cross sections. (4.16)

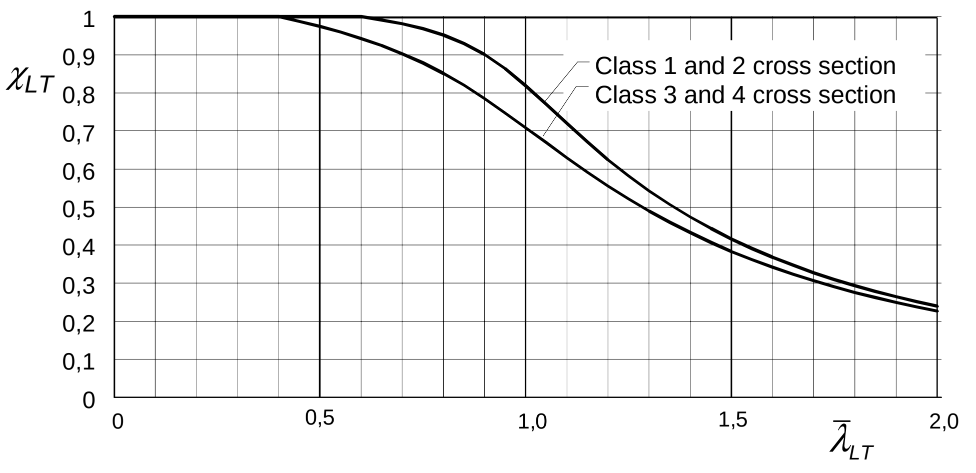
The value of $\bar{\lambda}_{LT}$ may be determined from

$$\bar{\lambda}_{LT} = \sqrt{\frac{\alpha W_{el,y} \, f_o}{M_{cr}}} \tag{4.17}$$

where:

α is taken from table 5.3 subject to the limitation $\alpha \leq W_{pl} / W_{el,y}$

M_{cr} is the elastic critical moment for lateral-torsional buckling, (see Annex H of Eurocode 9).



TALAT Training in Aluminium Application Technologies	Reduction factor χ_{LT} for lateral-torsional buckling	2301.04.15

The elastic critical moment for lateral-torsional buckling of a beam of uniform symmetrical cross-section with equal flanges, under standard conditions of restraint at each end, loaded through its shear centre and subject to uniform moment is given by:

$$M_{cr} = \frac{\pi^2 EI_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}} \tag{4.18}$$

where:

$$G = \frac{E}{2(1 + \nu)}$$

I_t is the torsion constant

I_w is the warping constant

I_z is the second moment of area about the minor axis

L is the length of the beam between points, which have lateral restraint.

The standard conditions of restraint at each end are:

- restrained against lateral movement
- restrained against rotation about the longitudinal axis
- free to rotate in plan

The elastic critical moment for lateral-torsional buckling of a beam of doubly symmetrical cross-section or mono-symmetric cross-section, under different conditions of restraint at the ends is found in Annex H of Eurocode 9.

5 Axial force

5.01 General

In this section, members subjected to axial compression or axial tension are treated.

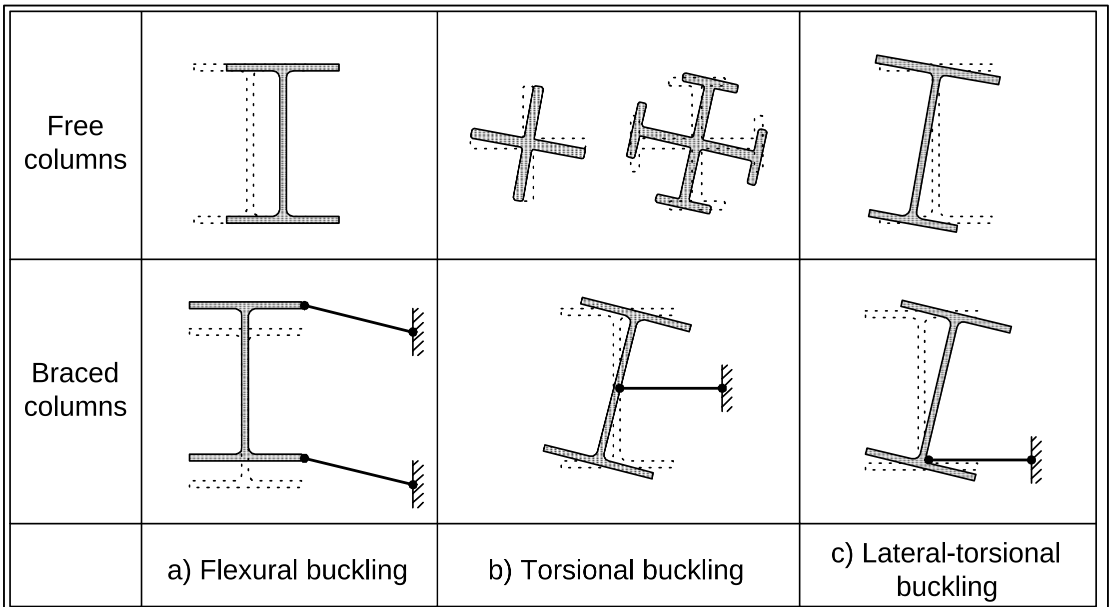
The resistance of a bar *in tension* is defined as the load where the mean stress is equal to the yield stress f_o/γ_{M1}. Failure in the structure will usually occur at a higher stress but then the deformation is so large that the structure no longer is of use. In certain cases of local weakening, such as bolt holes, small cut outs and welds, local yielding will occur without large deformations. Soon the stresses will reach the strain hardening range, and the resistance is governed by the ultimate stress f_a/γ_{M2}.

The term *local* as used above means a hole or a cut-out where the length parallel to the direction of the load is at the most 25% of the width of that part of the cross section.

The ultimate load for a bar *in compression* is usually determined by overall and/or local buckling.

Columns with a cross section symmetrical about both axes will buckle in one of the two principal planes (*flexural buckling* cf. **Figure 2301.05.01a**). If the torsional rigidity of the column is small, for example cruciform sections or angles, cf. **Figure 2301.05.01** b, *torsional buckling* may occur.

Beams with arbitrary (non-symmetrical) cross section may fail by *lateral-torsional buckling*, cf. **Figure 2301.05.01c**.



TALAT Training in Aluminium Application Technologies

Buckling modes for unbraced and braced compression members

2301.05.01

The buckling mode of a braced member depends on which axis is braced. The member may fail by buckling in a plane perpendicular to the one that is braced, by torsional buckling (if the braced axis passes through the shear centre) or by lateral-torsional buckling (any other plane), cf. **Figure 2301.05.01a-c**.

5.02 Tensile force

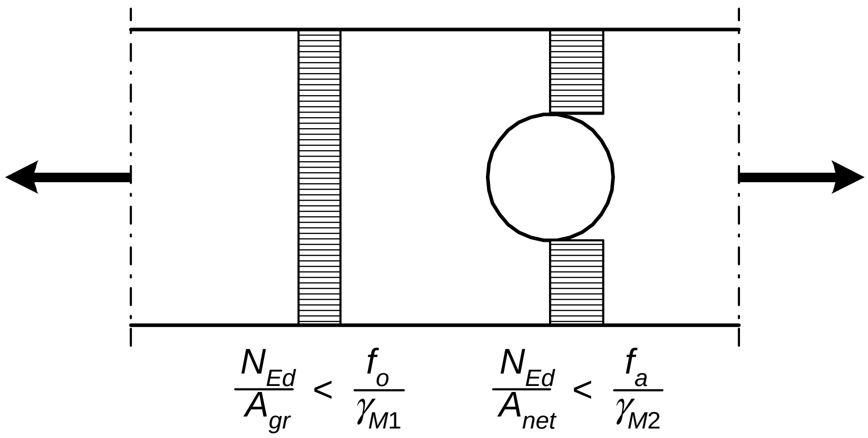
In the case of a member subjected to tensile force only, the resistance is given by smallest of the following two expressions:

$$N_{o,Rd} = A_g \frac{f_o}{\gamma_{M1}} \tag{5.1}$$

$$N_{a,Rd} = A_{net} \frac{f_a}{\gamma_{M2}} \tag{5.2}$$

where

A_{gr} = area of the gross cross section
A_{net} = net section area with deduction for holes and HAZ softening when required
f_o = characteristic value strength
f_a = characteristic ultimate strength
γ_{M1} = 1,1 = partial safety factor for yielding
γ_{M2} = 1,25 = partial safety factor for ultimate strength

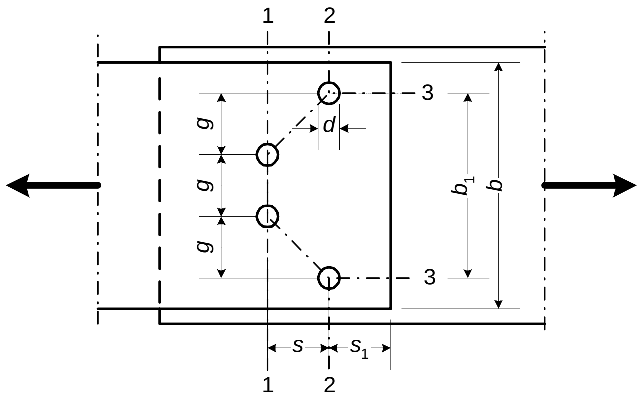


TALAT Training in Aluminium Application Technologies | Stresses in gross and net section | 2301.05.02

Expression 5.2 is valid for a local weakened section defined in 5.01. If the weakening is not to be considered as local, then A_{gr} is to be substituted with A_{net} in expression 5.1.

In the case of displaced holes as in **Figure 2301.05.03a**, different yield lines are considered, to find the smallest net area A_{net}. The width is reduced by the diameter of every hole that is passed, and increased by $s^2/4g$, but not more than 0.6 s. Here, the notation s is the distance between the centre of the holes parallel to the direction of the force and g is the distance perpendicular to the same direction.

The method can also be used for angle sections and other sections with bolt holes in more than one plane. The angle legs are then thought to be spread out and s and g are measured along the midline of the legs.



A_{net} is the smallest of

$t(b - 2d)$	in section 1
$t(b - 4d + 2s_2/(4g))$	in section 2
$t(b_1 - 4d + 2s_2/(4g) + 2\ 0{,}6s_1)$	in section 3

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Net section in section with displaced holes

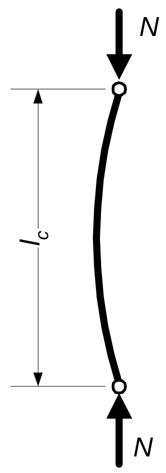
2301.05.03

As tension members are not exposed to buckling, they can be very slender. In order to avoid vibrations and deflections by unforeseen transverse loading, they should not be too slender. Often a limitation L/i < 240 is used for main structures and L/i < 300 for secondary structures, unless the tension members are pre-stressed.

5.03 Compressive force

This section is restricted to columns under centrally applied loads with no end moments. A column is a basic member of most structures, and knowledge of column behaviour is necessary in the interpretation and understanding of specification requirements (See **Figure 2301.05.04**).

The buckling strength of a column may be defined by the buckling load or the ultimate load. These definitions are applied to a column failure as distinct from local failure, such as local buckling of flanges or webs. But, as will be discussed later, local buckling will influence the overall buckling resistance. The buckling load, often called Euler buckling load N_E or elastic buckling load (critical load) N_{cr}, may be defined as that load at which the theoretically straight column assumes a deflected position. The ultimate load is the maximum load a column can carry. It marks the boundary between stable and unstable deflected positions of the column and is reached gradually, unlike the buckling load, which is an instantaneous phenomenon.



TALAT Training in Aluminium Application Technologies | Compresive force on a column | 2301.05.04

Buckling will occur only for a centrally loaded, perfectly straight column. The strength of practical columns, however, depends upon whether there are imperfections such as initial out-of-straightness, eccentricity of load, transverse load, end fixity, local buckling, non-linear σ–ε-curve, residual stresses or heat affected zones, HAZ. Most tests on columns did not isolate these various effects, and so a scatter band for column curves resulted because the maximum or ultimate load was observed, not the buckling load. The usual procedure for defining a column curve was much the same eighty years ago as now; the column curve was taken as the line of best fit through the test points, although modern calculation methods can explain, in many cases, the detailed behaviour if the imperfections are known.

To take into account the transition in the column curve from the Euler curve to the yield line, more or less complicated correction factors were involved, using estimated eccentricities, initial deflections or non-linear material curve. It has been shown that, for the hypothetical case of straight, centrally loaded, pin-end columns, the transition curve is due to, first of all, to the presence of residual stresses in the cross section. However, as residual stresses are small in extruded profiles, other imperfections such as non-linear σ–ε-curve and heat affected zones are of importance for aluminium columns.

5.031 Euler load, squash load and resistance

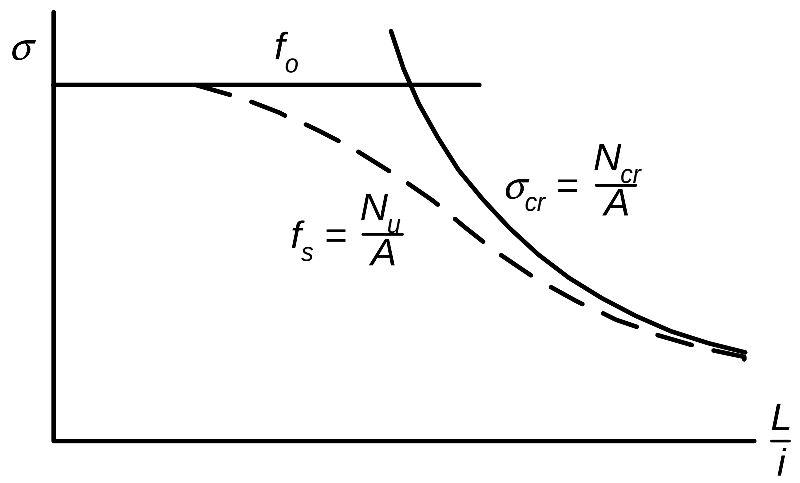
For a pin-ended column, perfectly straight and centrally loaded, the buckling load is known as the Euler load,

$$N_{cr} = \frac{\pi^2 EI}{L^2} \tag{5.3}$$

where N_{cr} is the Euler load, E the modulus of elasticity, I the second moment of area of the cross section and L is the length of the column between pin ends. When the column is not pinned-ended, an equivalent length of the column is used. Equivalent length (effective length, buckling length) is discussed later.

The strength of a column usually is indicated by a Acolumn curve≅, which shows graphically the relationship between the buckling stress and the corresponding slenderness ratio L/i, as shown in **Figure 2301.05.05**. Many empirical and semi-empirical column curves were developed. The corresponding expressions contain constants, which are a function of initial crookedness, residual stresses etc. As the residual stresses in steel sections are depending on cooling condition after rolling, which are different for different cross section shapes, a number of column curves for different cross section shapes are given in European specifications for steel. As the non-linearity of the σ–ε-curve and heat affected zones are more important for aluminium columns, two column curves are given in Eurocode 9 for aluminium columns and three coefficients η, k_1 and k_2 are added to the expression for resistance. These curves and coefficients allow for:

- Heat-treated alloys (α = 0,20 and λ_o = 0,1)
- Non heat-treated alloys (α = 0,32 and λ_o = 0,0)
- Local buckling ($\eta = A_{eff}/A$)
- Asymmetric cross section (k_1)
- Heat affected zones from longitudinal or transverse welds (k_2)



TALAT Training in Aluminium Application Technologies	Column curves	2301.05.05

For a member in compression the resistance is given by

$$N_{b,Rd} = \eta A \chi k_1 k_2 \frac{f_o}{\gamma_{M1}} \tag{5.4}$$

where

χ = a reduction factor for flexural buckling as given in 5.032 and for other modes of instability (torsional buckling and lateral-torsional buckling) as given in 5.036.

The coefficients k_1 and k_2 are given in the Eurocode. For symmetric cross sections and no welds k_1 = 1 and k_2 = 1

5.032 Reduction factor for flexural buckling

The reduction factor χ_c for flexural buckling depends on the type of aluminium alloy and the slenderness parameter according to expression 5.7.

The slenderness parameter λ_c (in Eurocode 9 denoted $\bar{\lambda}$) for a compression member is the square root of the squash load Af_o over the Euler buckling load N_{cr}

$$\lambda_c = \sqrt{\frac{A f_o}{N_{cr}}} \tag{5.5}$$

As $N_{cr} = \frac{\pi^2 EI}{l_c^2}$, then

$$\lambda_c = \sqrt{\frac{A f_o l_c^2}{\pi^2 E I}} = \frac{l_c}{\pi i}\sqrt{\frac{f_o}{E}} \quad \text{where } i = \sqrt{\frac{I}{A}} \tag{5.6}$$

A = area of the cross section (effective area to allow for local buckling)
E = characteristic value of modulus of elasticity
N_{cr} = elastic flexural buckling load (based on the gross cross section)
f_o = characteristic value of yield stress
i = radius of inertia
l_c = effective buckling length, see 5.035

The reduction factor χ is found in the diagram in **Figure 2301.05.06** or is calculated from

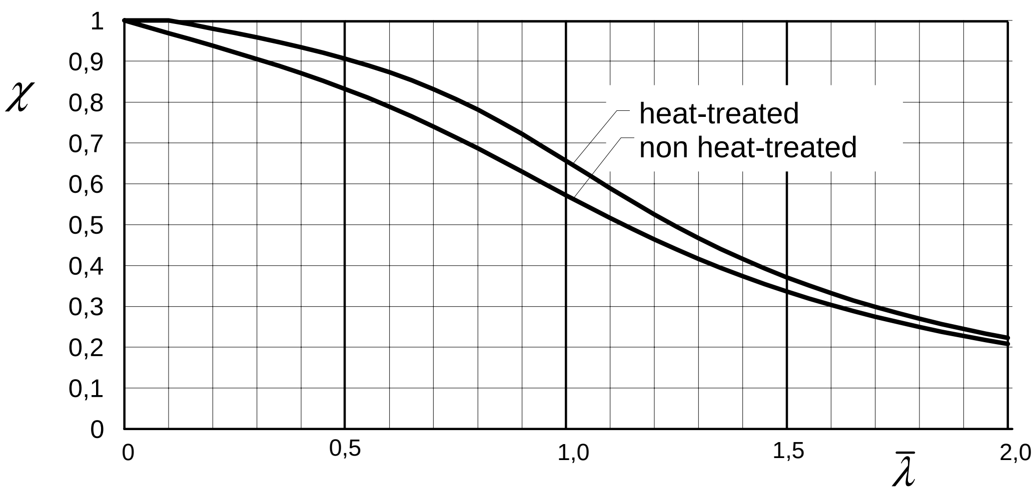
$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^2}} \leq 1{,}0 \tag{5.7}$$

where

$$\phi = 0{,}5(1 + \alpha(\lambda_c - \lambda_o) + \lambda_c^2) \qquad (5.8)$$

$\alpha = 0{,}20$ and $\lambda_o = 0{,}1$ for heat-treated alloys

$\alpha = 0{,}32$ and $\lambda_o = 0{,}0$ for non heat-treated alloys



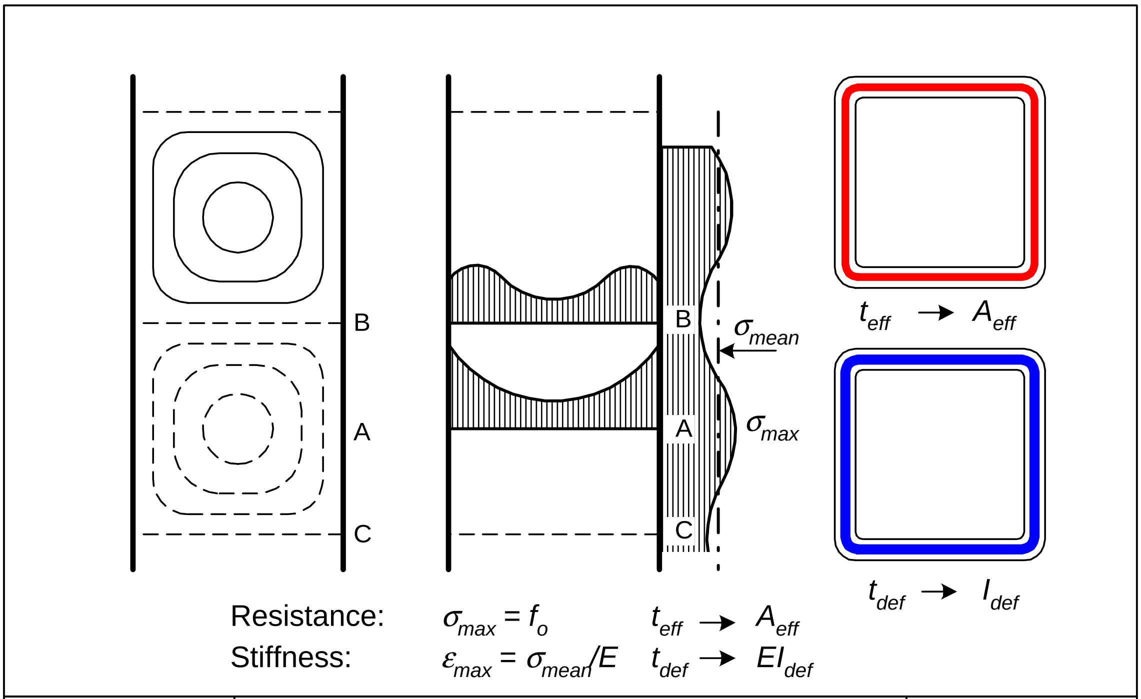
TALAT Training in Aluminium Application Technologies	Reduction factor χ for flexural buckling	2301.05.06

5.033 Cross section class 4

When slender parts of a cross section buckle locally, the bending *stiffness* along the column and the *resistance* of the cross section is reduced. The reduction of the stiffness can be allowed for by using an *effective cross section for stiffness*. Failure occurs when some cross sectional element reaches its local buckling *resistance*. This is allowed for by using an *effective cross section for resistance*, cf. section ***3:1***.

These two effective cross sections are not the same. The effective cross section for stiffness is normally larger than the effective cross section for strength, which is explained in conjunction to **Figure 2301.05.07**. The left figure shows a flange of a quadratic hollow section loaded in compression. The flange is buckled locally. As the section is quadratic, the length of the buckles is the same as the width of the flanges. In **Figure 2301.05.07** is also shown the stress distribution in a section through the middle of the buckle and also through the node line between two buckles. These stress distributions differ quite a lot in a slender flange. In the section through the buckle the stresses are largest at the borders. In the section between two buckles the stresses are levelled out to a certain extent. This means that the stresses (and the corresponding strains) vary along the column from a maximum at the centres of the buckles to a minimum at the nodes.

Of importance for the axial strength is the maximum stress. Of importance for the axial stiffness is the mean strain along the whole column. The effective area for stiffness is then a function of the mean strain along the column, which is proportional to the mean stress, whereas the effective area for strength is given by the maximum stress.



TALAT Training in Aluminium Application Technologies | Stress distribution in buckled plate and effective cross sections | 2301.05.07

One condition for the development of the stress distribution in the **Figure 2301.05.07** is that the longitudinal edges are free to deflect inwards when local buckling occurs. This is the case in a hollow section. In an inner panel of a wide plate with stiffeners, adjacent panels prevent the edges to deflect inwards. Then the stress distribution through the nodes is very similar to the stress distribution through the buckles. The maximum stresses are also smaller in a plate with straight edges than in a hollow section.

In calculation the effective area, A_{eff} is based on the effective thickness t_{eff} and the stiffness EI_{def} is based on another effective thickness t_{def}, $t_{def} > t_{eff}$. Based on A_{eff} and EI_{def} an effective radius of gyration is calculated

$$i_{eff} = \sqrt{\frac{I_{def}}{A_{eff}}} \tag{5.9}$$

to be used in the slenderness parameter

$$\lambda_c = \frac{l_c}{\pi\, i_{eff}} \sqrt{\frac{f_o}{E}} = \sqrt{\frac{f_o\, l_c^2\, A_{eff}}{\pi^2 E\, I_{def}}} = \sqrt{\frac{A\eta\, f_o\, l_c^2}{\pi^2 E\, I_{def}}} = \sqrt{\frac{A\eta\, f_o}{N_{cr}}} \tag{5.10}$$

The last square root is the formulation used in 5.9.4.1 in Eurocode 9. (Again note that λ_c is denoted $\bar{\lambda}$ in Eurocode 9). The strength of the column in compression is then given by

$$N_{b,Rd} = \chi A_{eff} \frac{f_o}{\gamma_{M1}} \tag{5.11}$$

where the reduction factor χ is determined according to 5.032 and the coefficients k_1 and k_2 are omitted.

The effective thickness t_{def} for a flat element supported along both edges parallel to the compressed member can be given by

$$t_{def} = 30t^2/b \le t \tag{5.12}$$

For a flat element with a free edge parallel to the compressed member, the effective thickness t_{def} can be given by

$$t_{def} = 12t^2/b \le t \tag{5.13}$$

The width b of the flat elements is to be calculated according to table 5.03. Note that t_{def} is independent of the strength of the material.

In most cases the slenderness b/t of the cross section elements is so small that the reduction of the stiffness can be ignored, which means that the gross cross section can be used when calculating the second moment of inertia $I_{def} = I_{gr}$.

5.034 Slenderness parameters

In summary, there are many different ways to calculate the *slenderness parameter* λ_c. In **Figure 2301.05.08** three ways are presented. They all give the same result. In the examples, methods 2 or 3 are preferred. As $\bar{\lambda}$ cannot be used in the Math-program used in the examples, λ_c is used instead. It is also difficult to use $\bar{\lambda}$ in some word processors. To distinguish between $\lambda = l/i$ and λ_c, λ is not used in the examples, but l/i which is called *slenderness ratio*.

Parameter	Eurocode 9	TALAT
slenderness parameter	$\bar{\lambda}$	λ_c
slenderness ratio	λ	l/i

Given:	$A = 4.2 \cdot 10^{3} \cdot mm^{2}$	$A_{eff} = 3.78 \cdot 10^{3} \cdot mm^{2}$	$f_o := 300 \cdot MPa$
	$I = 3.104 \cdot 10^{6} \cdot mm^{4}$	$L = 1.2 \cdot 10^{3} \cdot mm$	$E := 70000 \cdot MPa$
Alternative:	1. Eurocode 9	2. Definition	3. Radius i_{eff}
	$i := \sqrt{\frac{I}{A}}$	$N_{cr} := \frac{\pi^2 \cdot E \cdot I}{L^2}$	$i_{eff} := \sqrt{\frac{I}{A_{eff}}}$
	$\lambda := \frac{L}{i}$	$\lambda_c := \sqrt{\frac{A_{eff} \cdot f_o}{N_{cr}}}$	$\lambda_c := \frac{L}{i_{eff} \cdot \pi} \cdot \sqrt{\frac{f_o}{E}}$
	$\eta := \frac{A_{eff}}{A}$	$\lambda_c = 0.873$	$\lambda_c = 0.873$
	$\lambda_1 := \pi \cdot \sqrt{\frac{E}{\eta \cdot f_o}}$		
	$\lambda_c := \frac{\lambda}{\lambda_1}$		
	$\lambda_c = 0.873$		

TALAT alu Training in Aluminium Application Technologies	**Ways to calculate the slenderness parameter λ_c**	2301.05.08

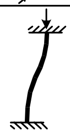
5.035 Buckling length

The effective buckling length l_c should be taken as *KL*, where *L* is the length between points of lateral support; for a cantilever strut, *L* is its length. The value of *K*, the effective length factor for struts, should be assessed from knowledge of the end conditions; table 5.01 gives guidance. The values of *K* take local deformation in joints and connections into account, which mean that the values are larger than the theoretical values for rigid restraints at the end. For example, the theoretical value is 0,5 for case no. 1 and 0,7 for case no. 2. In portal frames, case no 4 could be used if the stiffness of the beam is considerably larger than the stiffness of the column, say $I_{beam} > 10\ I_{column}\ L_{beam}\ /\ h_{column}$

With the formulation used in Eurocode 9, the value of *i* should be based on the gross section of the member. With the formulation in 5.033, i_{eff} should be used. The two formulations give at the end the same result.

According to Eurocode 9, when the cross section is wholly or substantially affected by HAZ softening at a directionally restrained end of a member, such restraint should be ignored in arriving at a suitable value for *K*. If due account is taken of the reduced strength in the heat-affected zone, end restrains can be utilised. See section 5.037. In welded frames, end restrains must be utilised in joints.

Table 5.01 Effective length factor *K* for struts

End condition		*K*
1. Held in position and restrained in direction at both ends		0,7
2. Held in position at both ends and restrained in direction at one end		0,85
3. Held in position at both ends, but not restrained in direction		1.0
4. Held in position at one end, and restrained in direction at both ends		1,25
5. Held in position and restrained in direction at one end, and partially restrained in direction but not held in position at the other end		1,5
6. Held in position and restrained in direction at one end, but not held in position or restrained at the other end		2,0

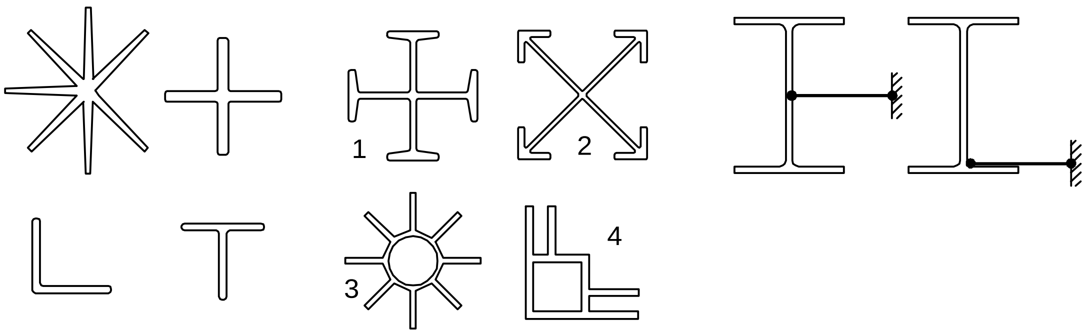
5.036 Torsional buckling and lateral-torsional buckling

For members with small torsion stiffness such as angles, tees and cruciform sections, torsional buckling has to be considered.

The possibility of torsional buckling may be ignored for the following:

a) Closed hollow sections
b) Double symmetrical I-sections, unless braced
c) Sections composed entirely of radiating outstands, e.g. angles, tees, cruciforms, that are classified as class 1 cross sections

For sections such as angles, tees and cruciforms, composed entirely of radiating outstands, local and torsional buckling are closely related. When considering the torsional buckling of sections containing only unreinforced outstands, allowance should be made, where appropriate, for the presence of HAZ material when determining A_{eff} but no reduction should be made for local buckling, i.e. $\rho_c = 1$.



a) "Outstand" sections   b) "General" cross sections   c) Braced columns

TALAT — Training in Aluminium Application Technologies | Cross sections susceptible to torsional buckling | 2301.05.09

For sections containing reinforced outstands such that mode 1 would be critical in terms of local buckling (see 5.4.3 in Eurocode 9), the member should be regarded as "general" and A_{eff} determined allowing for either or both local buckling and HAZ material. The effective thickness for outstands in unsymmetric sections (see e.g. Figure 5.12b4 in Eurocode 9) shall have the reduced value according to 5.4.5(3) b) in Eurocode 9.

The slenderness parameter λ_c is given by

$$\lambda_c = \sqrt{\frac{f_o A}{N_{cr}}} \qquad (5.14)$$

where

N_{cr} = the elastic critical load for (lateral-)torsional buckling

The value of χ for torsional buckling should be obtained from the expression given in 5.8.4.1 (1) in Eurocode 9 with α and λ_o in ϕ determined from

$\alpha = 0{,}35$ and $\lambda_o = 0{,}4$ — for "general" cross-sections, which are cross-sections other than composed entirely of outstands, see **Figure 2301.05.09b**, and for the (lateral-) torsional buckling check of braced columns, see **Figure 2301.05.09c**

$\alpha = 0{,}20$ and $\lambda_o = 0{,}6$ — for cross-sections composed entirely of radiating outstands, see **Figure 2301.05.09a**.

5.037 Design of splices and end connections

In the ultimate limit state, a compressed element is subjected to bending moment of the first order increased by the bending moment due to the normal force multiplied by the deflection. These additional moments must be taken into consideration when designing splices and end connections which are often weaker than the element itself.

End connections and splices shall have the same strength as the remainder of the member, or be designed for the concentric compressive force *N*, the bending moment caused by transverse loads and an additional bending moment ΔM

$$\Delta M = \frac{N \alpha W_{el}}{A_{ef}} \left(\frac{1}{\chi} - 1 \right) \sin \frac{\pi x_s}{l_c} \tag{5.15}$$

where

A_{ef} = area of effective cross-section

W_{el} = modulus of effective cross-section in bending

χ = reduction factor

x_s = distance between a point of contra flexure in buckling and the splice or end connection

l_c = buckling length

Splices and connections should be designed in such a way that load can be transmitted to the effective portions of the cross-section.

When the structural details at the ends of a member are such that there is doubt regarding the point of load introduction, a suitable eccentricity shall be assumed in design.

Joints that are presumed to be simply supported must be designed so that maximum angular deflection is possible without failure.

5.04 Welded columns and columns with columns with bolt holes or cut-outs

5.041 Longitudinal welds

Welded members assembled by longitudinal welds contain residual stresses and have reduced strength in the heat affected zones (**Figure 2301.05.10**)

The method for calculation is the same as for members in cross sectional class 4 with the difference that the effective area is reduced by

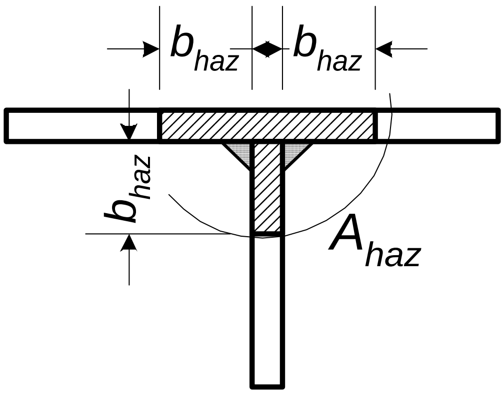
$$\Delta A = A_{haz} \, (1 - \rho_{haz}) \tag{5.16}$$

where

A_{haz} = area of the heat-affected zone, HAZ. The extent of HAZ is given in 5.5.3 in Eurocode 9.

ρ_{haz} = HAZ reduction factor given in 5.5.2 in Eurocode 9

Furthermore, in a welded column the coefficient k_2 is less than 1,0, see Table 5.5 in Eurocode 9.



TALAT Training in Aluminium Application Technologies

Heat affected zone

2301.05.10

5.042 Transverse welds

The influence of transverse welds on the buckling resistance depends, to some extent, on the location of the welds. If a weld is close to the mid-section of the column, the buckling strength is about the same as if the whole column was made of heat-affected material. If the transverse welds are close to pinned ends, they do not reduce the buckling strength, but the squash load. This means that, if the slenderness of the column is large and the buckling resistance is small compared to the squash load, the transverse welds do not influence the resistance.

In Eurocode 9 the coefficients k_2 is used to take the influence of transverse welds into consideration. The simple way is to conservatively use $k_2 = \rho_{haz}$. Less conservative is using (see **Figure 2301.05.11**)

$$k_2 = \frac{\rho_{haz} \; f_a/\gamma_{M2}}{f_o/\gamma_{M1}} \; \frac{1}{\chi + (1-\chi)\sin(\pi x_s / l_c)} \quad \text{but } k_2 \le 1{,}0 \qquad (5.17)$$

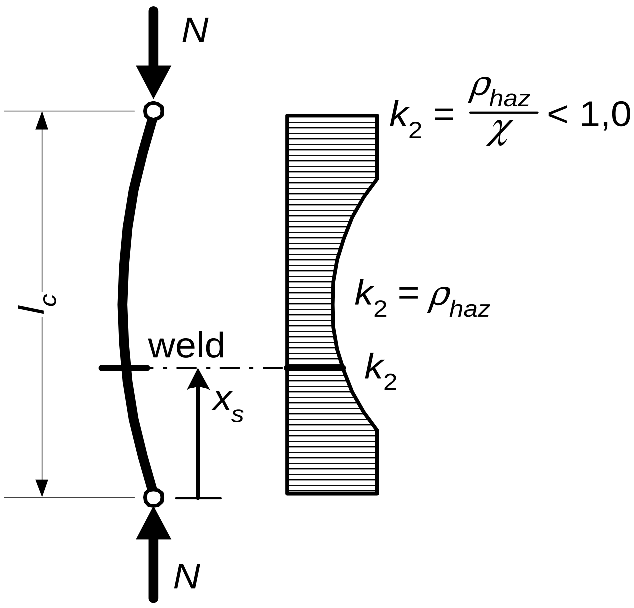
where

ρ_{haz} = the HAZ reduction factor

χ = reduction factor for flexural buckling

x_s = the distance from the section with the weld under consideration to a point of contra-flexure in the deflection curve for buckling. If the column is pin-ended, x_s is measured to one end, no matter which one.

l_c = buckling length



TALAT Training in Aluminium Application Technologies	Reduction factor for transverse weld	2301.05.11

5.043 Columns with unfilled bolt-holes or cut-outs

The expression 5.17 is used, except that

ρ_{haz} is replaced by A_{net}/A_{gr} (5.18)

where

A_{net} = net section area, with deduction of holes

A_{gr} = gross section area

5.05 Built-up members

Battened struts should generally be designed by first determining the forces to which each component will be subjected and then proportioning each component to resist these forces. However, providing the arrangement satisfies the seven conditions in 5.8.8 in Eurocode 9 it may be designed as a single compound member. In the following design rules for the case that not all of these seven conditions are fulfilled are given.

A built-up member is defined as a lattice column consisting of two or more longitudinal members connected together as in **Figure 2301.05.11a-d**, or as a member with batten plates as in **Figure 2301.05.11e**.

The interaction between longitudinal elements in a built-up member is not complete. In the case of buckling, this may be taken into consideration by increasing the slenderness. The initial value is the slenderness parameter λ_o that the column would have in the case of complete interaction.
For a laced column the slenderness parameter λ_o is increased to

$$\lambda_c = \sqrt{\lambda_0^2 + \frac{f_o}{E}\left(\frac{A d^3}{A_d\, a b^2} + \frac{Ab}{A_b\, a}\right)} \qquad (5.19)$$

for a member according to **Figure 2301.05.11 a**

$$\lambda_c = \sqrt{\lambda_0^2 + \frac{f_o\, A d^3}{E\, A_d\, a b^2}} \qquad (5.20)$$

for a member according to **Figure 2301.05.11 b, c and d**

where

A = sum of the cross section area of the longitudinal members
i = the radius of inertia in the case of complete interaction
A_d = the total cross section area of the diagonal members in the plane parallel to the buckling plane
A_b = the total cross section area of the transverse elements in the plane parallel to the buckling plane
a = the distance between points as given in **Figure 2301.05.11 a-d**
d = the length of the diagonal elements

$$\lambda_0 = \frac{l_c}{i\pi}\sqrt{\frac{f_o}{E}}$$

For a column with rigid batten plates as in **Figure 2301.05.11e** the value of l_c is

$$\lambda_c = \frac{l_c}{\pi}\sqrt{\frac{f_o}{E}\left(\frac{l_c^2}{i^2} + \frac{a^2}{i_1^2}\right)} \quad \text{if} \quad \frac{I_1 b}{I_b a} < \frac{1}{3} \qquad (5.21)$$

where

i = radius of inertia for the cross section of the longitudinal element in the case of complete interaction
a = distance between two batten plates as in **Figure 2301.05.11e**
b = distance between the centre of mass of two longitudinal elements in the buckling

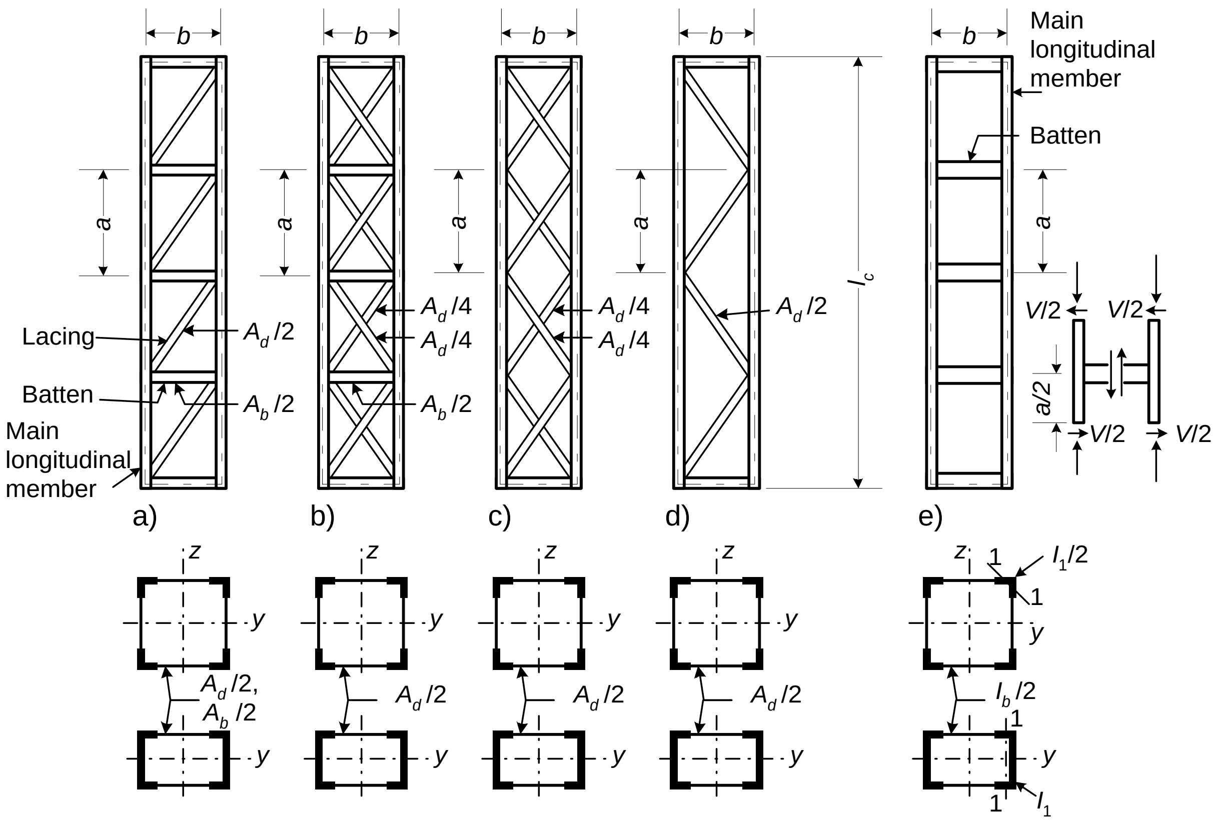
plane

i_1 = smallest radius of inertia for one of the longitudinal elements

I_1 = sum of the smallest values of moment of inertia for two longitudinal elements, type 1, in **Figure 2301.05.11e**, the moment of inertia for one longitudinal element, type 2, in **Figure 2301.05.11e**

I_b = sum of the moments of inertia for the cross sections of the batten plates

l_c = buckling length



TALAT (Training in Aluminium Application Technologies)	a) – d) Laced built-up column. e) Battened built-up column	2301.05.12

Expression 5.11c is valid under the condition that the intersections between the diagonal and the transverse elements and the longitudinal elements are rigid. This means that the connections are welded or bolted with prestressed bolts.

Each individual element and joint shall be designed for a shear force by external loading plus ta a fictitious transverse load q_x.

$$q_\chi = \frac{0{,}015\, N_{Ed}}{l_c \left(1 - \dfrac{N_{Ed}\, \lambda_c^2}{0{,}9\, A\, f_o} \right)} > 0 \qquad (5.22)$$

where N_{Ed} = the applied axial force.

For a built-up column with batten plates, the inflection points for the deflection curve of the longitudinal elements, are assumed to be half way between the batten plates. The shear force in these points, cf. **Figure 2301.05.11e**, results in bending moments in the longitudinal elements.

Square lattice columns built up by angle sections bolted together, may be designed as follows.

The buckling length for a diagonal element, fixed with one bolt at each end, is the distance between the bolts. In the case of two bolts in both ends, the buckling length is that between the closest bolts. It is assumed that the element will buckle in the weakest direction.

The eccentricity of the force is taken into consideration by means of an effective slenderness parameter λ_{ef} as follows.

$$\lambda_{ef} = 0{,}6 + 0{,}57\,\lambda_c \qquad \text{if} \quad \lambda_c < 1{,}4 \qquad (5.23)$$

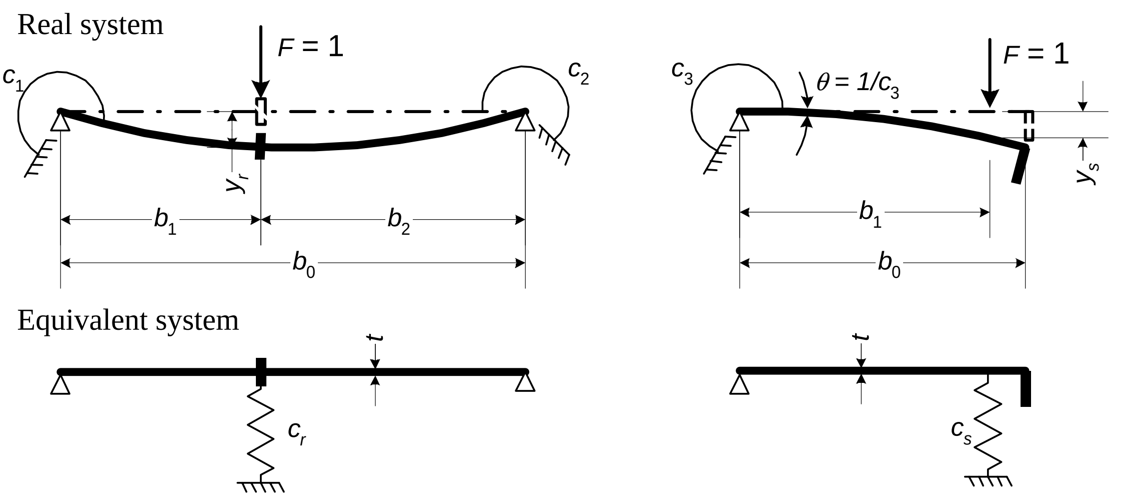
$$\lambda_{ef} = \lambda_c \qquad \text{if} \quad \lambda_c \geq 1{,}4 \qquad (5.24)$$

The buckling length of the main elements is equal to the distance between the bolts that fix the diagonal element on one side, if they coincide with the fixing points on the adjacent side. It is assumed that the bar will buckle in the weakest direction. If the points on the adjacent side are displaced by half the distance between the points then it is assumed that buckling will occur in the same plane as the sides.

5.06 Elements with edge or intermediate stiffeners in compression

5.061 General

Figure 2301.05.13. Determination of spring stiffness.



Spring stiffness

$$c_r = \frac{1}{y_r} \quad \text{where} \qquad\qquad c_s = \frac{1}{y_s} \quad \text{where}$$

$$y_r = \frac{4(1-\nu^2)b_1^2 b_2^2}{E\,t^3(b_1 + b_2)} \qquad\qquad y_s = \frac{4(1-\nu^2)b_1^3}{E\,t^3} + \phi\ b_1$$

if $c_1 = c_2 = 0$

ϕ is the rotation of adjacent parts of the cross section caused by a spring moment $1 \cong b_1$

TALAT Training in Aluminium Application Technologies	Determination of the spring stiffness for intermediate and edge stiffeners	2301.05.13

A direct method is given in 5.064 for elements reinforced with single-sided ribs or rip of thickness equal to the element thickness. In 5.062 - 5.063, a more general procedure is presented.

Design of stiffened elements is based on the assumption that the stiffener itself acts as a beam on elastic foundation, where the elastic foundation is represented by a spring stiffness depending on the transverse bending stiffness of adjacent parts of plane elements and on the boundary conditions of these elements. Determination of the spring stiffness is illustrated in **Figure 2301.05.13** for intermediate and edge stiffeners respectively.

The resistance of the stiffener is given by

$$N_{r,R} = \chi\, f_o A_r \qquad (5.25)$$

where

χ = buckling coefficient, to be determined according to 5.032 for a slenderness λ_c (see below) with the imperfection factor α = 0,2 and λ_o = 0,6

A_r = area of effective cross section composed of the stiffener and half the adjacent plane elements (see 5.062 and 5.063)

$$\lambda_c = \sqrt{\frac{f_o A_r}{N_{r,cr}}} \tag{5.26}$$

where $N_{r,cr}$ is the buckling load of the stiffener given by

$$N_{r,cr} = 2\sqrt{c E I_r} \qquad \text{if} \quad l \geq \pi \sqrt[4]{\frac{EI_r}{c}} \tag{5.27}$$

$$N_{r,cr} = \frac{\pi^2 E I_r}{l^2} + \frac{l^2 c}{\pi^2} \qquad \text{if} \quad l < \pi \sqrt[4]{\frac{E I_r}{c}} \tag{5.28}$$

where

c = spring stiffness

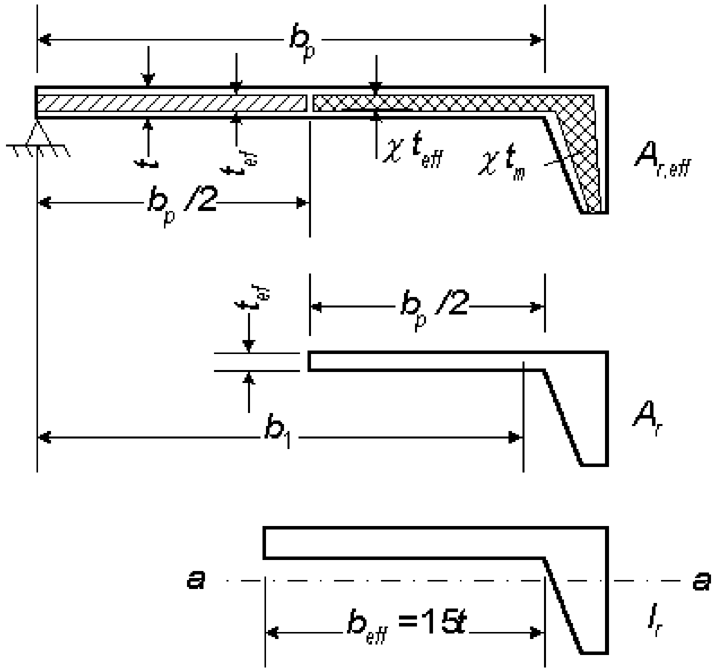
I_r = second moment of area of an effective cross section composed of the stiffener itself and part of adjacent plane elements with the effective width 15*t*. Compare 5.052.

l = spacing between rigid transverse stiffeners

5.062 Edge stiffeners

Edge stiffeners or edge folds shall only be considered as supports to the plane element, if the angle between the stiffener or fold and the plate deviates from the right angle by not more than 45^o and if the width of the edge stiffener is larger than 0,2b_p (with b_p according to **Figure 2301.05.14**). Otherwise, they should be ignored.

The effective area A_{ref} of edge stiffeners according to **Figure 2301.05**.14 may be estimated as follows:



TALAT	Edge stiffener, notations and effective cross sections for A_r and I_r	2301.05.14

Step 1: Determination of the effective edge section with an effective thickness of the plate element assuming the plate to be rigidly supported along both longitudinal edges.

Step 2: Consideration of the elastic support of the plate element by determining the buckling stress of the edge section, considered being elastically supported as follows:

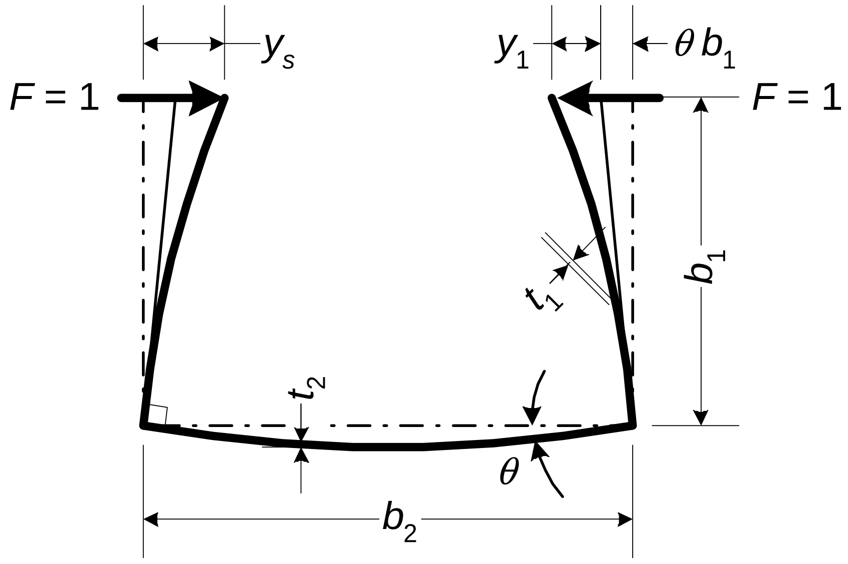
- Determination of effective thickness t_{ef} and effective width b_{def} = 15t according to 5.033 of the stiffener and the adjacent plane cross sectional element, cf. **Figure 2301.05.14**. (Step 1)

- The effective cross-sectional area A_r is given by using the effective thickness t_{ef}, cf. **Figure 2301.05.16 a and b**

- The second moment of area I_r is based on a cross section with effective width b_{def} = 15t for internal element and 12t for outstands, referred to the neutral axis a-a of the effective edge section.

- The critical buckling load N_{cr} of the edge stiffener is given by formula 5.27 or 5.28.

- The reduced effective area A_{ref} then will be $A_{ref} = \chi A_r$ with χ according to 5.032, expressed by a second reduction of the thickness to $\chi\, t_{ef}$ or $\chi\, t_{av}$ where t_{av} = average thickness of edge stiffener.



TALAT Training in Aluminium Application Technologies

Channel section

2301.05.15

For the channel section in **Figure 2301.05**.15 the deflection y_s is given by

$$y_s = y_1 + \theta\, b_1 = \frac{F\, b_1^3\, 12(1-\nu^2)}{3\, E\, t_1^3} + \frac{F\, b_1^2\, b_2\, 12(1-\nu^2)}{2\, E\, t_2^3} \qquad (5.29)$$

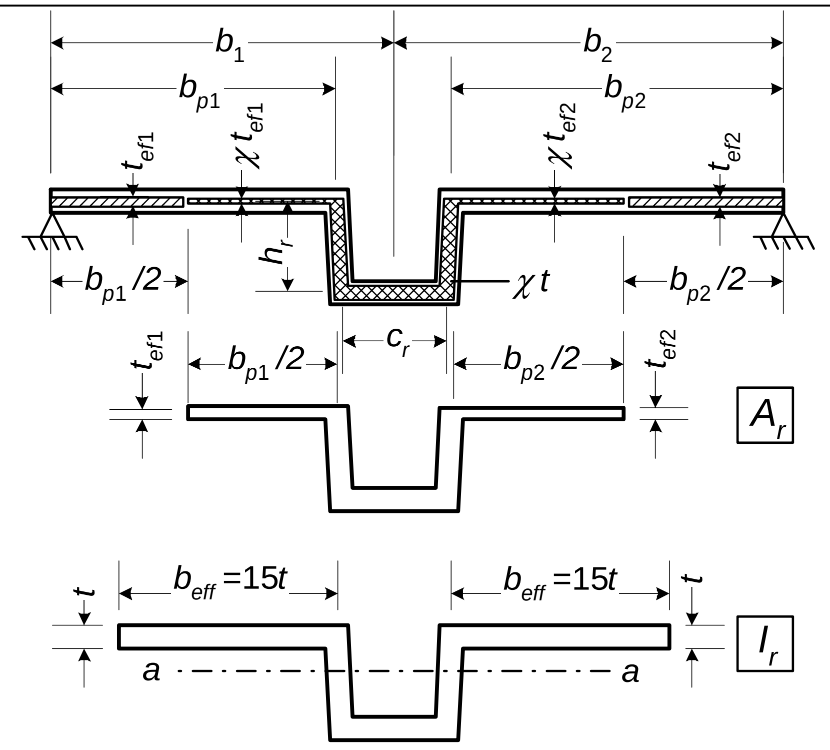
Expression 5.27, with $c = F/y_s$, then gives:

$$N_{s,cr} = 1{,}05\, E \sqrt{\frac{I_s\, t_1^3}{b_1^3 \left(1 + \dfrac{1{,}5\, b_2\, t_1^3}{b_1\, t_2^3}\right)}} \qquad (5.30)$$

for a section without transverse stiffeners.

5.063 Intermediate stiffeners

The design rules given below are valid for elements, which are supported along both edges. The stiffener cross section incorporates the stiffener itself plus the effective portions of adjacent parts of the section. Stiffeners may be grooves or extrusions. The effective section is described in **Figure 2301.05.16**. The validity of the design formula is limited to two equally shaped stiffeners at maximum.



Intermediate stiffeners; notations and effective cross section for A_r and I_r — 2301.05.16

The effective area A_{ref} of intermediate stiffeners according to **Figure 2301.05.16** may be estimated as follows:

Step 1: Determination of the effective parts of the stiffened element, assuming that the stiffeners provide effective supports to the adjacent plate elements.

Step 2: Consideration of the elastic support of the plate element by determining the buckling stress of the stiffener area, considered being elastically supported as follows:

- Determination of the effective thickness t_{ef} and effective width b_{def} of the element (Step 1) of the stiffener and the adjacent plane elements.

- The effective cross-sectional area A_r is based on the effective thickness t_{ef}. The moment of inertia is based on the effective width b_{def} and is referred to the neutral axis a-a of the stiffener section.

- The critical buckling load of the intermediate stiffener with an area A_r is given by formula 5.27 or 5.28 for a spring stiffness according to **Figure 2301.05.13**.

- The reduced effective area A_{ref} then will be:

$$A_{ref} = \chi A_r \tag{5.31}$$

with χ according to 5.061.

For a section with a groove as in **Figure 2301.05.16**

$$N_{r,cr} = 1{,}05E \frac{\sqrt{I_r t^3 s_1}}{b_{p1}(s_1 - b_{p1})} \tag{5.32}$$

where

$$s_1 = b_{p1} + b_{p2} + 2h_r + c_r \tag{5.33}$$

and the thickness is constant along the cross section.

5.064 Direct method for single-sided rib or lip

When the reinforcement is a single-sided rib or lip of thickness equal to the element thickness *t*, the equivalent slenderness β of the plate element is

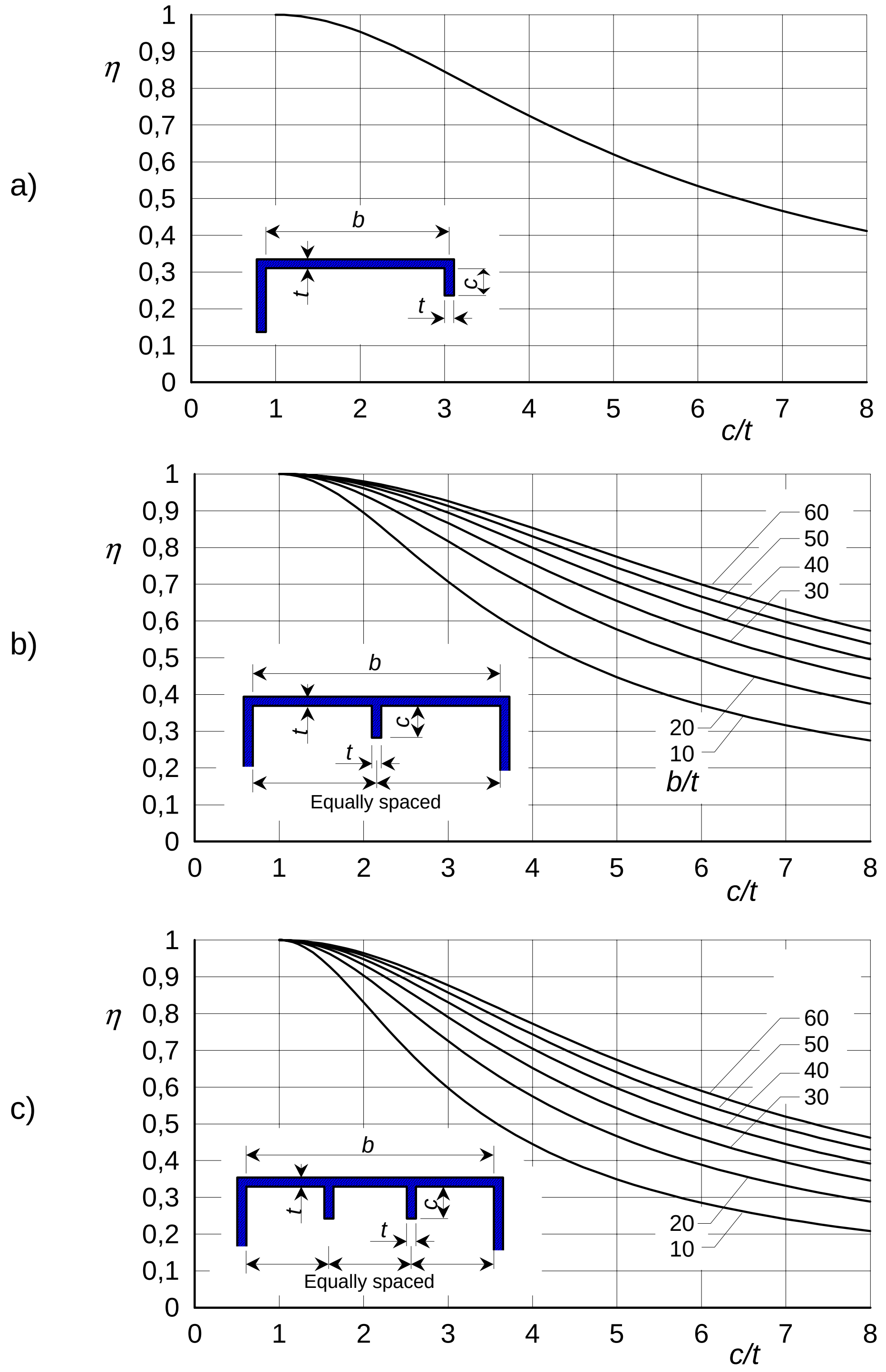
$$\beta = \eta \frac{b}{t} \tag{5.34}$$

where η is given in expressions 5.35, 5.36 or 5.37, or is read from **Figure 2301.05.16 a, b or c**. In this figure the depth *c* of the rib or lip is measured to the inner surface of the plate element.

$$\eta = \frac{1}{\sqrt{1 + 0{,}1(c/t - 1)^2}} \quad \text{(Figure 2301.05.17 a)} \tag{5.35}$$

$$\eta = \frac{1}{\sqrt{1 + 2{,}5\dfrac{(c/t - 1)^2}{b/t}}} \quad \text{(Figure 2301.05.17 b)} \tag{5.36}$$

$$\eta = \frac{1}{\sqrt{1 + 4{,}5\dfrac{(c/t - 1)^2}{b/t}}} \quad \text{(Figure 2301.05.17 c)} \tag{5.37}$$



TALAT Training in Aluminium Application Technologies

Values of η for reinforced elements

2301.05.17

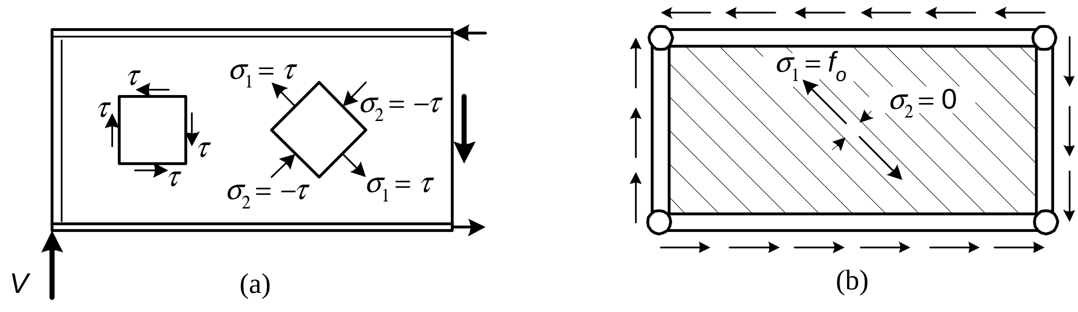
5.07 Multi-stiffened plates and orthotropic plates

The design principles presented in 5.06 are also being used for multi-stiffened plates and orthotropic plates. See 5.11.6 in Eurocode 9. Examples are presented in 5.08.

6 Shear force

6.01 Shear buckling of plate girder webs

For webs in shear there is a substantial post-buckling strength provided that, after buckling, tension membrane stresses, anchored in surrounding flanges and transverse stiffeners, can develop. In a pure shear state of stress the magnitude of the principal membrane stresses σ_1 and $-\sigma_2$ are the same as long as no buckling has occurred ($\tau < \tau_{cr}$, see **Figure 2301.06.01a**). After reaching the buckling load ($V_{cr} = \tau_{cr}\, h_w\, t_w$) the web will buckle and redistribution of stresses start. Increased load results in increased tensile stress σ_1 but only slight, if any, increase in the compressive stress σ_2.



TALAT Training in Aluminium Application Technologies | a) Pure shear state of stress. b) Ideal tension field | 2301.06.01

For the extreme case where the flanges are completely prevented to move towards each other by an external structure, rigid in the plane of the web, a so-called *ideal tension field* can develop without any compression stresses. For a slender web, the shear force resistance is then given by

$$V_w = \frac{\sqrt{3}}{2}\, f_v\, h_w\, t_w \tag{6.01a}$$

where $f_v = f_o/\sqrt{3}$ is the yield stress in shear. This is only 13% less than the resistance of a web that does not buckle.

In an I-shaped or a box-shaped girder the flanges can normally move freely towards each other, except close to transverse stiffeners. When the spacing between transverse stiffeners is large, the edges can move freely within a large portion of the girder. The web in shear must then carry both tensile and compressive stresses after buckling, but the shear force can exceed the buckling load $V_{cr} = \tau_{cr}\, h_w\, t_w$ because of redistribution of stresses after buckling involving increased tensile membrane stresses in the web.

The web can carry membrane tensile stresses in the longitudinal direction but in the transverse direction no membrane tensile stresses can occur. This results in a state of membrane stresses according to **Figure 2301.06.02h** with stress trajectories in an angle of 45° along the flanges where there are no web deflections and less than 45° in the middle of the web. Compare stress trajectories in **Figure 2301.06.03**. The resistance can be given by the theory of the *rotated stress field* which, for slender webs gives a shear resistance of approximately

$$V_w \approx \frac{1{,}32}{\lambda_w} f_v h_w t_w \qquad (6.01b)$$

where

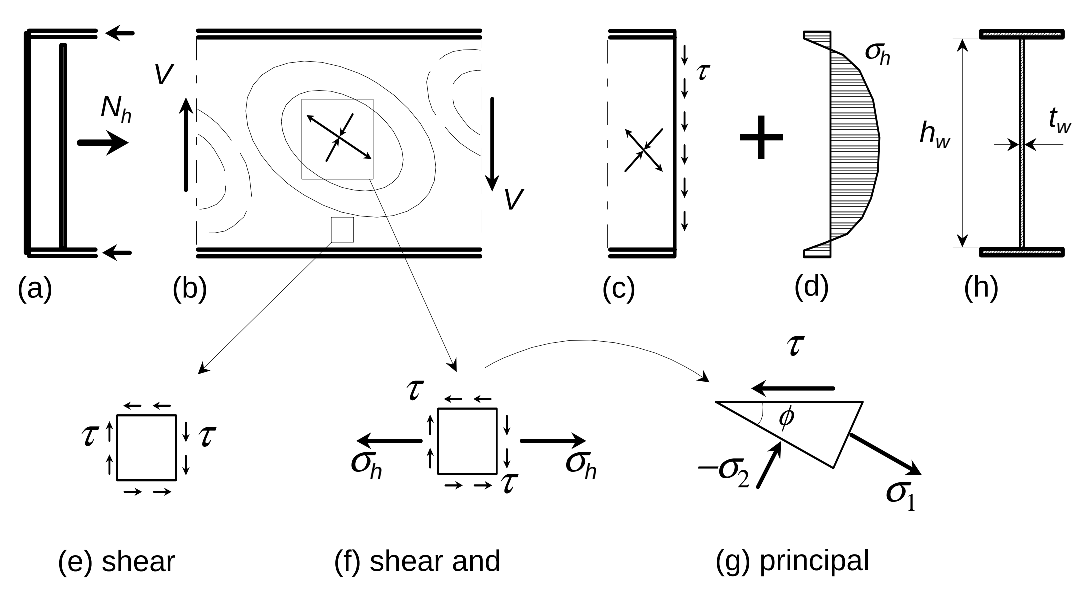
$$\lambda_w = \sqrt{\frac{f_v}{\tau_{cr}}} \qquad (6.01c)$$

is a slenderness parameter. For a web panel with large aspect ratio a/h_w the shear buckling stress τ_{cr} is

$$\tau_{cr} = k_\tau \frac{\pi^2 E}{12(1-\nu^2)} \frac{t_w^2}{b_w^2} \qquad (6.01d)$$

where

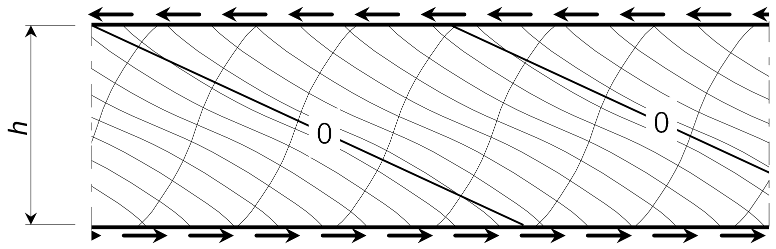
$$k_\tau = 5{,}34 \qquad (6.01e)$$



TALAT Training in Aluminium Application Technologies | Stresses in a web in the post-buckling range | 2301.06.02

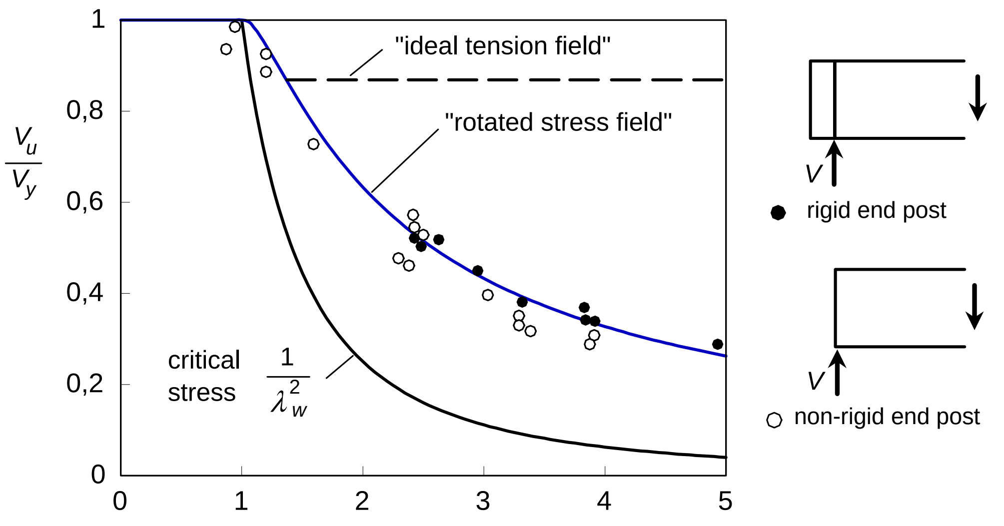
Experiments carried out for steel and aluminium girders with stiffeners at supports only, showed good agreement with this theory, see **Figure 2301.06.04**. In the figure you can see that there is a large post buckling range for slender webs ($1/\lambda_v^2$ corresponds to the buckling load).

One condition for tension fields to be developed is that longitudinal tensile stress, see **Figure 2301.06.02d**, are anchored at the girder ends. This is possible by pairs of stiffeners, which, together with the web, create a transverse beam with the flanges as supports. See **Figure 2301.06.02a**, *rigid end post.*



TALAT Training in Aluminium Application Technologies	**Stress trajectories for $\tau = 3\tau_{cr}$**	2301.06.03

Transverse stiffeners prevent web buckling locally and prevent the flanges from moving towards each other, if the flanges are rigid and the transverse stiffener spacing is not too large (a/h_w less than about 3). The shear strength depends upon the distance between the transverse stiffeners and the lateral stiffness of the flanges. For example the shear strength of a slender web with closely spaced transverse stiffeners is close to the formula for ideal tension field.

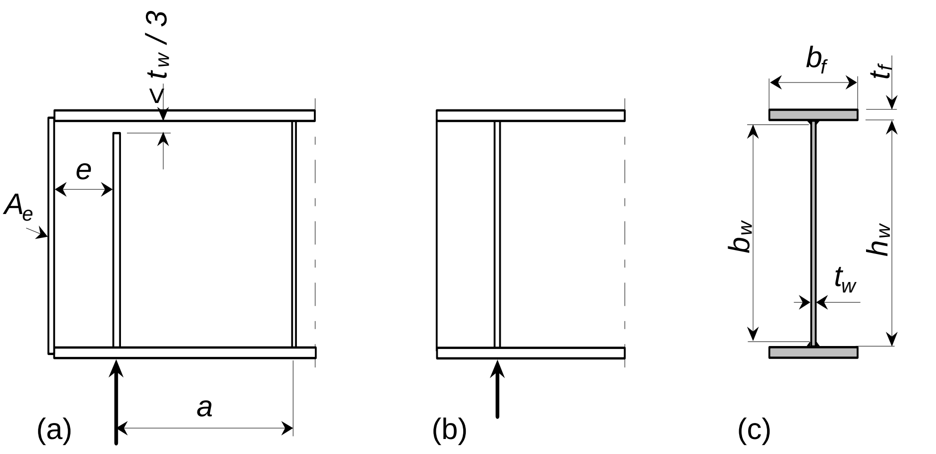


TALAT Training in Aluminium Application Technologies | Comparisons between tests and theories | 2301.06.04

When the slenderness is small, the shear force resistance is larger than that corresponding to the yield strength f_v in shear. This is because the shear deformation can become large without buckling and strain hardening can occur, resulting in $\tau_u > f_v$ and $\eta > 1/\sqrt{3}$ = 0,577, see table 6.01. The shear resistance is then a function of the ultimate strength f_{uw} of the web material.

Thick flanges can further increase the shear resistance after the formation of plastic hinges in the flanges and increased tension in the web. See Eurocode 9. This increase, $V_{f,Rd}$, is of minor importance in ordinary I-girders and can usually be omitted. Compare the design example in 6.05.

6.02 Shear resistance of webs with stiffeners at supports only



TALAT Training in Aluminium Application Technologies	**a) Rigid end post b) Non rigid end post c) Cross section**	2301.06.05

The shear resistance of a web of a girder is the lesser of V_{wd} allowing for local buckling and V_{ad} allowing for failure in a net section where

$$V_{wd} = \rho_v h_w t_w \frac{f_o}{\gamma_{M1}} \qquad (6.02a)$$

$$V_{ad} = 0{,}58\, A_{w,net} \frac{f_a}{\gamma_{M2}} \qquad (6.02b)$$

where

h_w = web depth inclusive fillets, see 6.05c

b_w = depth of flat part of a web, see **Figure 2301.06.05c**

t_w = web thickness, see **Figure 2301.06.05c**

A_{wn} = the net area of a cross section through bolt holes in the web

ρ_v = reduction factor determined from table 6.01 or **Figure 2301.06.06**.

In table 6.01 and **Figure 2301.06.06** a distinction is made between a rigid end post and a non-rigid end post.

A rigid end post stand for

- an end web panel with double stiffeners at end support
- a web panel not at the end of the girder, inclusive
- a web panel adjacent to an inner support of a continuos girder

Non-rigid end post is applicable for

- an end web panel of a girder with a single stiffener at end support

- a web panel of a box girder in torsion where all sides have similar ratio b_w / t_w
- a local panel of a trapezoidal web, see 6.04

Table 6.01 Reduction factor ρ_v for shear buckling

λ_w	Rigid end post	Non-rigid end post
$\lambda_w \geq 0{,}48/\eta$	η	η
$0{,}48/\eta < \lambda_w < 0{,}949$	$0{,}48/\lambda_w$	$0{,}48/\lambda_w$
$0{,}949 \leq \lambda_w$	$1{,}32/(\lambda_w + 1{,}66)$	$0{,}48/\lambda_w$

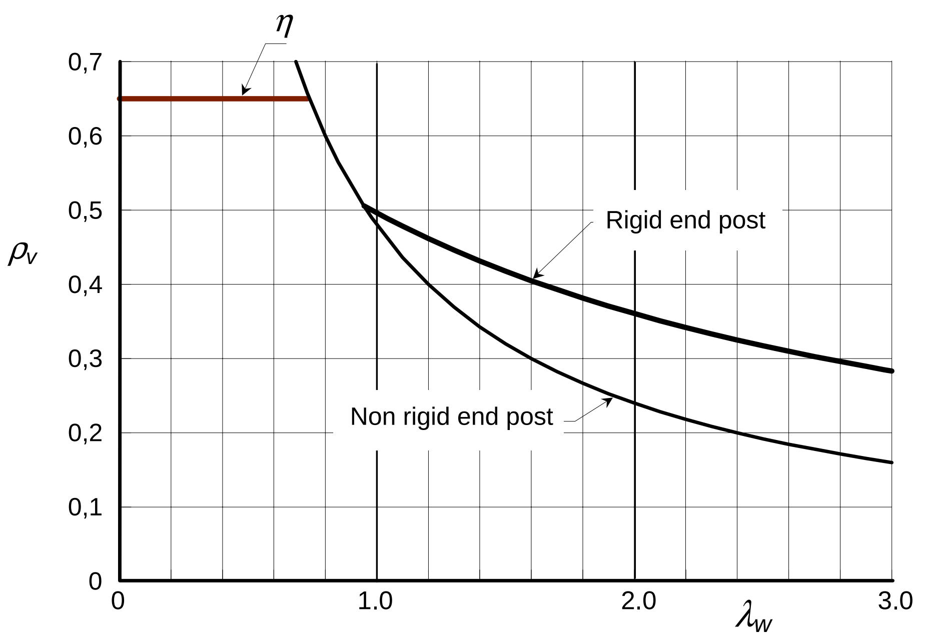
$\eta = 0{,}4 + 0{,}2\, f_{uw} / f_{ow}$ but less than 0,7 where
f_{ow} is the characteristic strength for overall yielding and f_{ow} is the characteristic ultimate strength of the web material.

In table 6.01 and **Figure 2301.06.06** the slenderness parameter λ_w of the web is

$$\lambda_w = 0{,}35 \frac{b_w}{t_w} \sqrt{\frac{f_o}{E}} \qquad (6.02c)$$

The following conditions apply to a rigid end post, see **Figure 2301.06.05a**

$$A_e > 4 b_w t_w^2 / e \text{ och } e > 0{,}1 b_w \qquad (6.02d)$$



TALAT Training in Aluminium Application Technologies | Reduction factor for shear buckling | 2301.06.06

6.03 Plate girders with intermediate stiffeners

For a plate girder with longitudinal and transverse stiffeners, the reduction factor ρ_v from table 6.01 or **Figure 2301.06.06** is used. The value of the slenderness parameter λ_w is

$$\lambda_w = \frac{0{,}81}{\sqrt{k_\tau}} \frac{b_w}{t_w} \sqrt{\frac{f_o}{E}} \tag{6.03a}$$

but not less than corresponding to the largest sub-panel, depth b_{w1}

$$\lambda_w \geq \frac{0{,}81}{\sqrt{5{,}34 + 4\,(b_{w1}/a)^2}} \frac{b_{w1}}{t_w} \sqrt{\frac{f_o}{E}} \tag{6.03b}$$

where

$$k_\tau = 5{,}34 + \frac{4{,}00}{(a/b_w)^2} + k_{\tau st} \qquad \text{if } a/b_w \geq 1{,}00 \tag{6.03c}$$

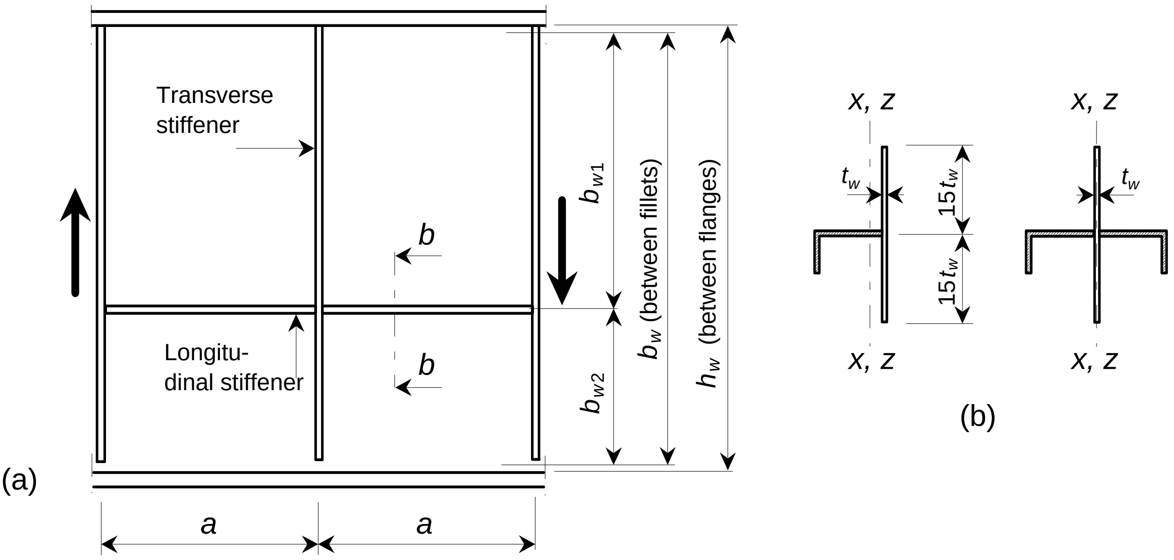
$$k_\tau = 4{,}00 + \frac{5{,}34}{(a/b_w)^2} + k_{\tau st} \qquad \text{if } a/b_w < 1{,}00 \tag{6.03d}$$

$$k_{\tau st} = 9\left(\frac{b_w}{a}\right)^2 \left(\frac{I_{st}}{t_w^3 b_w}\right)^{\frac{3}{4}} \quad \text{but not less than} \quad \frac{2{,}10}{t_w} \sqrt[3]{\frac{I_{st}}{b_w}} \qquad (6.03e)$$

I_{st} = second moment of area for the area of the cross section of the longitudinal stiffener with respect to the x-axis, see **Figure 2301.06.07b**. In the case of more than one stiffener of the same size, I_{st} is the sum of the second moment of area for each.

b_{w1} = width of the largest sub panel of the web, see **Figure 2301.06.07**

a = distance between the transverse stiffeners.



TALAT Training in Aluminium Application Technologies | a) Web with stiffeners. B) Cross section of stiffeners | 2301.06.07

Intermediate transverse stiffeners acting as rigid supports for the web panels should have a second moment of area I_{st} with respect to the x-axis for a cross section composed of the stiffener(s) and a strip of $30t_w$ of adjacent web plate, see **Figure 2301.06.07b**, fulfilling the following:

$$I_{st} \geq 1{,}5 \, \frac{h_w^3 \, t_w^3}{a^2} \quad \text{if} \quad \frac{a}{h_w} < \sqrt{2} \qquad (6.03f)$$

$$I_{st} \geq 0{,}75 \, h_w \, t_w^3 \quad \text{if} \quad \frac{a}{h_w} \geq \sqrt{2} \qquad (6.03g)$$

The strength of an intermediate stiffener should be checked for an axial compressive force equal to the actual shear force V_{Ed} minus the shear resistance

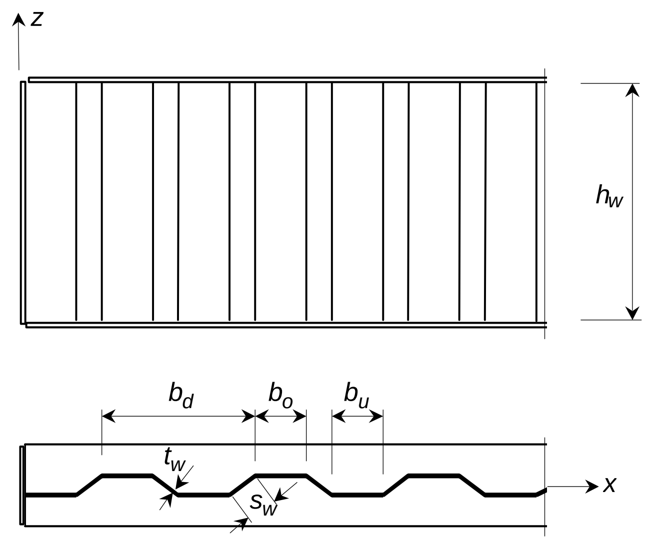
$V_{R0} = \rho_v b_w t_w f_{ow}$ of the web with the considered stiffener removed. As, on the safe side, $\rho_v = 0{,}48 / \lambda_w$ then V_{R0} approximately is

$$V_{R0} = 1{,}4 t_w^2 \sqrt{E f_{ow}} / \gamma_{M1} \qquad (6.03h)$$

Stiffeners that do not transfer loads to the tension flange may end at a distance from the flange equal to three times the thickness of the web, see **Figure 2301.06.05a**.

6.04 Corrugated or closely stiffened webs

In plate girders with transverse reinforcement in the form of corrugations or closely spaced transverse stiffeners ($a/b_w < 0{,}3$) the flat parts between stiffeners can buckle locally, and the transverse reinforcement may deform with the web in an overall buckling mode.



TALAT Training in Aluminium Application Technologies	Corrugated web	2301.06.08

The shear force resistance with respect to local shear buckling of flat parts is

$$V_{w,Rd} = 0{,}7 \rho_v t_w h_w f_{ow} / \gamma_{M1} \qquad (6.04a)$$

where

ρ_v is found in table 5.12 for $\lambda_w = 0{,}35 \dfrac{b_m}{t_w} \sqrt{\dfrac{f_o}{E}}$

b_m is equal to the greatest width of the corrugated web plate panels, b_o, b_u or s_w.

The shear force resistance with respect to global shear buckling is determined according to

$$V_{o,Rd} = \chi_o \, h_w \, t_w \, \frac{f_o}{\gamma_{M1}} \tag{6.04b}$$

where:

$$\chi_o = \frac{0{,}60}{0{,}8 + \lambda_{ow}^2} \quad \text{but not more than } 0{,}6 \tag{6.04c}$$

$$\lambda_{ow} = \sqrt{\frac{h_w \, t_w \;\; f_o}{V_{o,cr}}} \tag{6.04d}$$

$$V_{o,cr} = \frac{60\,E}{h_w} \, \sqrt[4]{I_z \;\; I_x{}^3} \tag{6.04e}$$

$$I_z = \frac{b_d}{b_u + b_o + 2\,s_w} \, \frac{t_w^3}{10{,}9} \tag{6.04f}$$

I_x is the second moment of area of the corrugated web per unit width, see Figure 05.25 in Eurocode 9 .

Calculation examples are given in 11.6

7 Concentrated loads and support reactions

7.01 Beam webs without stiffeners

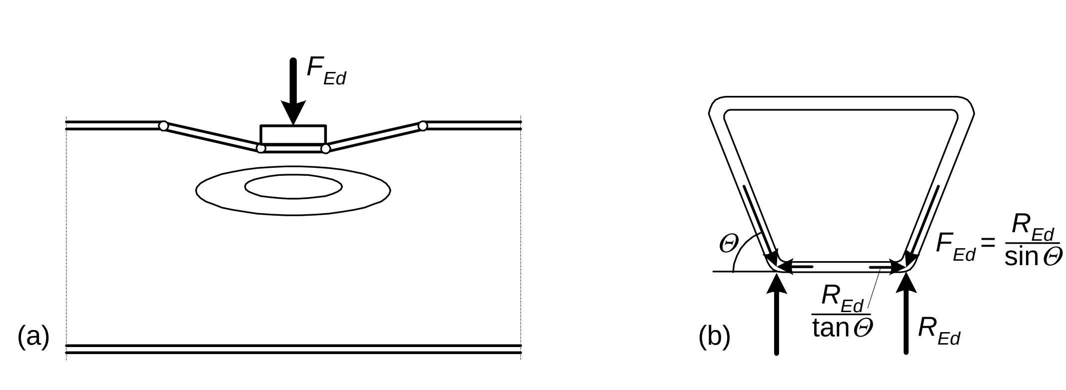
The resistance of an unstiffened web to transverse forces applied through a flange, is governed by one of the following modes of failure

- crushing of the web close to the flange, accompanied by plastic deformation of the flange

- crippling of the web in form of localized buckling and crushing of the web close to the flange, accompanied by plastic deformation of the flange

- buckling of the web over most of the depth of the member

- overall buckling of the web over a large part of the length of the member. This failure mode is most likely to take place when there are a number of transverse forces or a distributed load along the member.



TALAT — a) Buckling of the web and plastic hinges in the flanges under a concentrated load, b) Forces in inclined webs — 2301.07.01

A distinction is made between three types of load application, as follows:

a) Forces applied through one flange and resisted by shear forces in the web, *patch loading*, see **Figure 2301.07.03(a).**

b) Forces applied to one flange and transferred through the web directly to the other flange, *opposite patch loading*, see **Figure 2301.07.03(b).**

c) Forces applied through one flange close to an unstiffened end, *end patch loading*, see **Figure 2301.07.03(c).**

For box girders with inclined webs the resistance should be checked for both web and flange. The load effects are the components of the external load in the plane of the web and flange respectively. See **Figure 2301.07.01.** The effect of the transverse force on the moment resistance of the member should be considered.

The resistance of a longitudinally stiffened web is increased due to the presence of the stiffeners but no information is given in Eurocode.

The design approach in Eurocode 9 follows the same procedure as for other buckling cases. The resistance F_R is given as a reduction factor χ times the yield resistance F_y, where the reduction factor is a function of the slenderness parameter

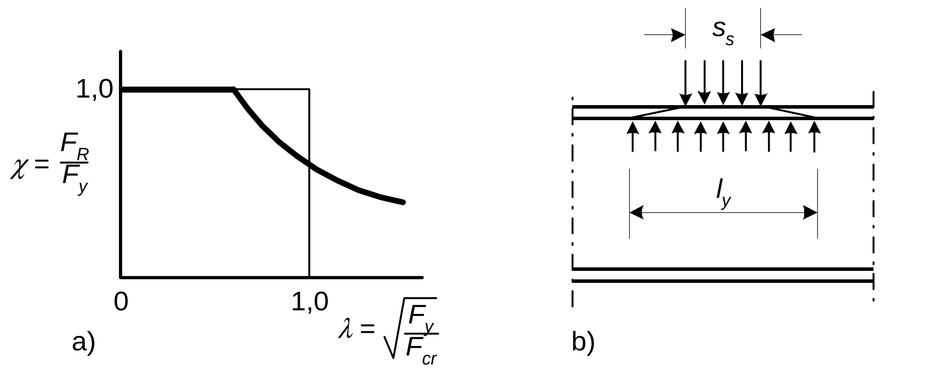
$$\lambda = \sqrt{\frac{F_R}{F_{cr}}} \quad (7.01)$$

where F_{cr} is the elastic buckling load. As there are few tests available on patch loading of aluminium beams and plate girders the following simple expression is used for χ , see **Figure 2301.07.02 a**).

$$\chi = \frac{0,6}{\lambda} \text{ but not more than } 1,0 \quad (7.02)$$

Then

$$F_R = \chi F_y = 0,6\sqrt{F_{cr} F_y} \text{ but not more than } F_y \quad (7.03)$$



TALAT Training in Aluminium Application Technologies | **a) Reduction factor χ for b) patch loading** | 2301.07.02

The elastic buckling load is

$$F_{cr} = k_F \frac{\pi^2 E}{12(1-\nu^2)} \frac{t_w^3}{b_w} \quad (7.04)$$

where k_F is a coefficient depending on loading condition.

The yield load for patch loading can be expressed as

$$F_y = l_y t_w f_y \qquad (7.05)$$

where l_y is an effective loading length, depending on actual loading length s_s and stiffness of loaded flange. See **Figure 2301.07.02 b**).

The resulting resistance

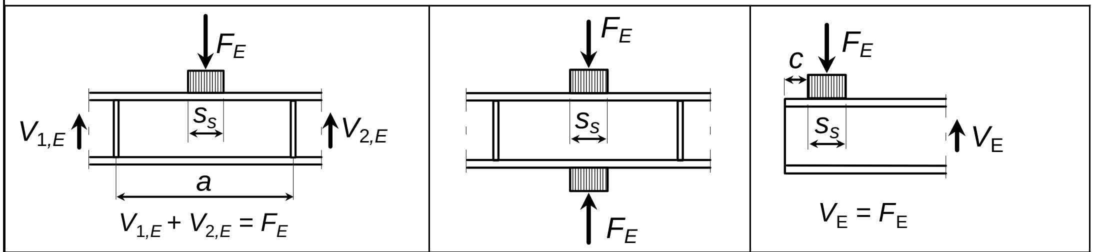
$$F_R = 0{,}6 \sqrt{\frac{k_F \pi^2 E t_w^3}{12(1-\nu^2) b_w} l_y t_w f_y} \quad \text{but not more than} \quad l_y t_w f_y \qquad (7.06)$$

Introducing f_{ow} instead of f_y and the resistance factor γ_{M1} gives the design resistance F_{Rd} for a transverse force (**Figure 2301.07.03(a), (b) and (c)**) from

$$F_{Rd} = 0{,}57\, t_w^2 \sqrt{\frac{k_F l_y f_{ow} E}{b_w}} \frac{1}{\gamma_{M1}} \quad \text{but not more than} \quad t_w l_y \frac{f_{ow}}{\gamma_{M1}} \qquad (7.07)$$

where

f_{ow} is the characteristic strength of the web material and
k_F is given in **Figure 2301.07.03**.



application a)	application b)	application c)
$k_F = 6 + 2\left(\frac{b_w}{a}\right)^2$	$k_F = 3{,}5 + 2\left(\frac{b_w}{a}\right)^2$	$k_F = 2 + 6\,\frac{s_s + c}{b_w} \le 6$

(7.08)

TALAT Training in Aluminium Application Technologies — Load applications and buckling coefficients — 2301.07.03

The length of stiff bearing, s_S, on the flange is the distance over which the applied force is effectively distributed and it may be determined by dispersion of load through solid material at a slope of 1:1, see **Figure 2301.07.04**. s_S should not be taken as more than b_W.

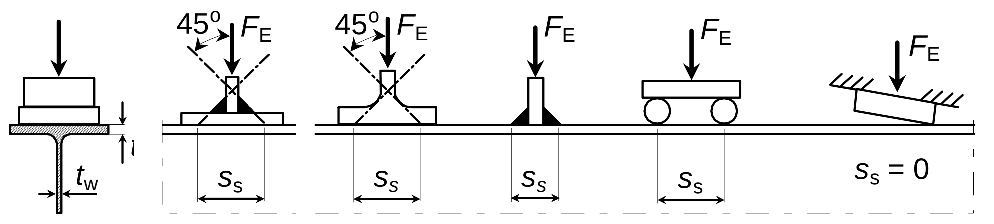
If several concentrated loads are closely spaced, the resistance should be checked for each individual load as well as for the total load. In the latter case s_s, should be taken as the centre distance between the outer loads.

The effective loaded length l_y, is calculated using the dimensionless parameters

$$m_1 = \frac{f_{of} \; b_f}{f_{ow} t_w} \tag{7.09}$$

$$m_2 = 0{,}02\left(\frac{b_w}{t_f}\right)^2 \quad \text{if} \quad \frac{(s_s + 4t_f)b_w \; f_{ow}}{k_F \, E \, t_w^2} > 0{,}2 \quad \text{else} \quad m_2 = 0 \tag{7.10}$$

For box girders, b_f in expression (7.09) is limited to $25t_f$ on each side of the web.



TALAT Training in Aluminium Application Technologies

Length of stiff bearing

2301.07.04

For load application a) and b) in **Figure 2301.07.03**, l_y is given by

$$l_y = s_s + 2t_f \left(1 + \sqrt{m_1 + m_1}\right) \tag{7.11}$$

For load application c) in **Figure 2301.07.02**, l_y is given by the smaller of expressions (7.11), (7.13) and (7.14). Note that $s_s = 0$ if the loading device does not follow the change in the slope of the girder end.

$$l_{ef} = \frac{k_F \, E \, t_w^2}{2 \, f_{ow} b_w} \leq s_s + c \tag{7.12}$$

$$l_y = l_{ef} + t_f \sqrt{\frac{m_1}{2} + \left(\frac{l_{ef}}{t_f}\right)^2 + m_2} \tag{7.13}$$

$$l_y = l_{ef} + t_f \sqrt{m_1 + m_2} \qquad (7.14)$$

7.02 Beam webs with stiffeners

Transverse stiffeners at supports and under concentrated loads are normally placed in pairs on both sides of the web, and are attached to the loaded flange.

The stiffener is designed as a compressed member (column) with a cross section composed of two stiffeners and a strip of the web having a width of $25t_w$ at interior stiffeners and $12t_w$ at the ends of the beam.

If both flanges are supported laterally, the stiffener member is supposed to have a buckling length equal to $0{,}8b_w$. If only one flange is supported laterally, the buckling length of the bar will depend on the connecting structure, but with a minimum value of $1{,}6b_w$. Consideration must also be taken to any lateral reaction from the unsupported flange.

If an end post is the only means of providing resistance against twist at the end of a girder, the second moment of area of the end-post section about the centre-line of the web (I_{ep}) should satisfy:

$$I_{ep} \geq b_w^2 \, t_f \, R_{Ed} / 250 W_{Ed} \qquad (7.15)$$

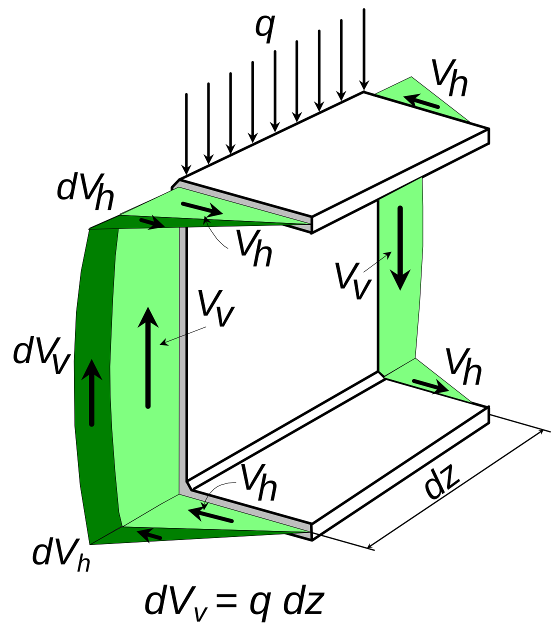
where

t_f	is the maximum value of flange thickness along the girder
R_{Ed}	is the reaction at the end of the girder under factored loading
W_{Ed}	is the total factored loading on the adjacent span.

8 Torsion

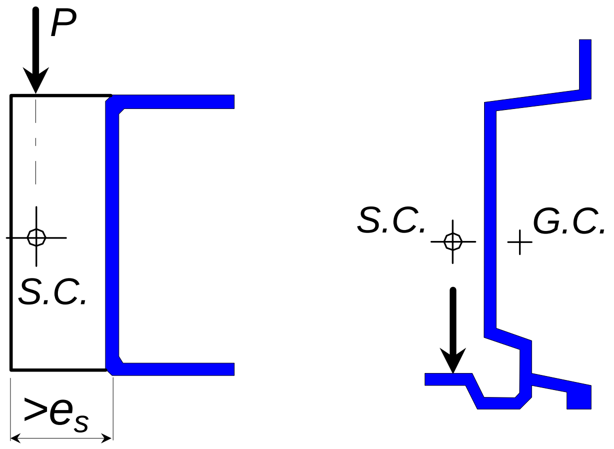
8.01 Shear centre

Aluminium extrusions are usually shaped to serve many different purposes and functions. They have therefore often complicated cross sections which are not symmetrical. The shear centre, SC, will then not coincide with the gravity centre, G.C. If the transverse load acts outside the shear centre, the beam will be acted upon by a torsional moment. For example, the shear centre of a channel section is located outside the web. See **Figure 2301.08.01**. When a load is acting on the top of the flange, the beam will be loaded by a torsional moment $F_{ed}\, e_s$ where e_s is the distance from the shear centre to the midline of the web.



TALAT Training in Aluminium Application Technologies	Shear flow and shear centre	2301.08.01

To avoid torsion, or to reduce the stresses as a result of torsion, you can (see **Figure 2301.08.02** and **Figure 2301.08.03**):



a. Add stiffener

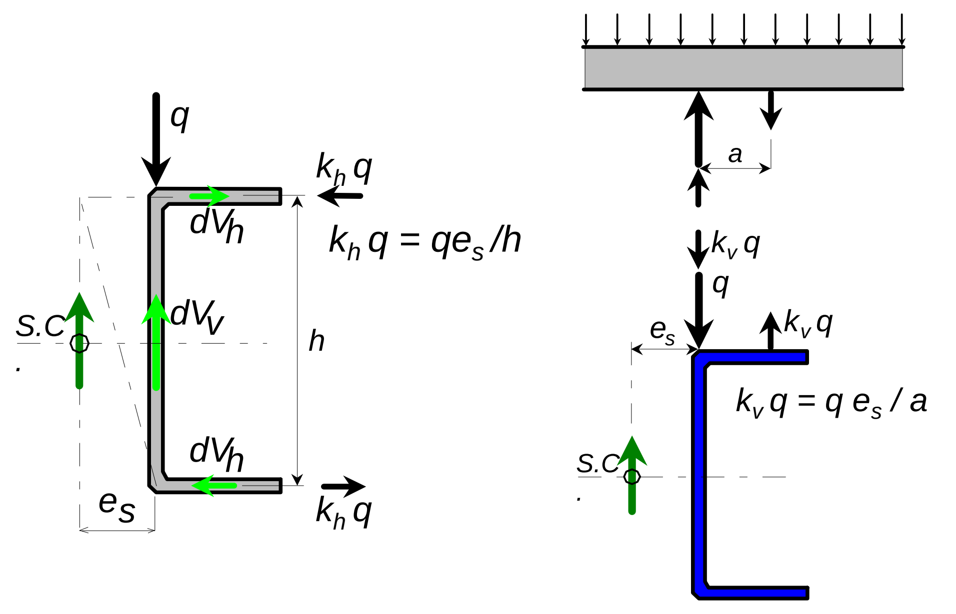
b. Shape the beam to bring the applied force through the shear centre

Design to avoid torsion	2301.08.02

Move the load to a point above the shear centre by adding a stiffener outside the web;
Design the beam so the load can act below the shear centre;

Add lateral bracing to the flanges. Bracing forces: $k_h\, q = qe_s/h$;

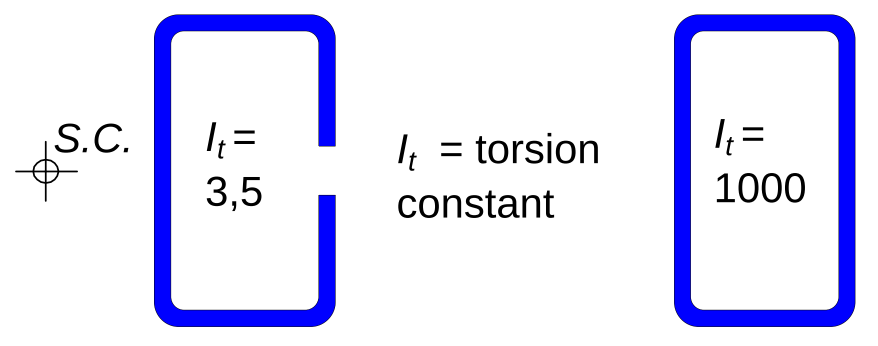
Attach the beam to the loading slab. Vertical forces: $k_v\, q = qe_s/a$;
Use a hollow section, which have much larger torsional stiffness than an open cross section.
Calculation of the position of the shear center can be tricky. In Annex J of Eurocode 9 the shear center position and the warping factor is given for some common thin-walled sections. In the examples in 11.02, a method for the calculation of cross section constants of arbitrary thin-walled open cross sections is given.



TALAT	Forces to prevent torsion	2301.08.03

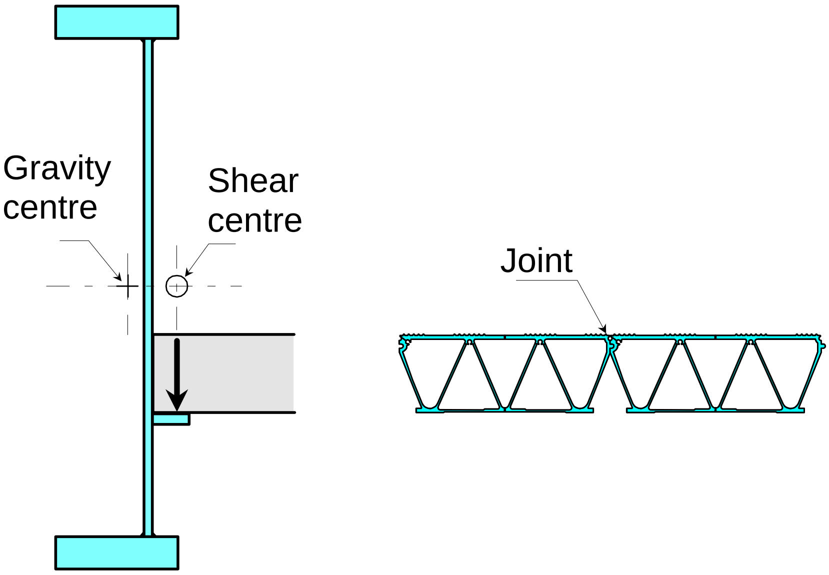
8.02 Closed and open sections

The extrusion technique makes it possible to produce hollow sections with large torsional stiffness. See **Figure 2301.08.04**. This should be made best use of in design, for instance in deck structures. In **Figure 2301.08.05** a deck built up by hollow section elements joined together with groove and tongue is shown. The deck profiles are multi-cell hollow sections supported by two welded I-beams with the web displaced such a distance that the shear center ends up above the supporting stiffener of the deck elements. In **Figure 2301.08.06** are shown the bending moment diagrams for deck elements loaded by concentrated wheel loads from a vehicle. Bending moments for three different deck profiles are given:



TALAT	Torsion constant of open and closed section	2301.08.04

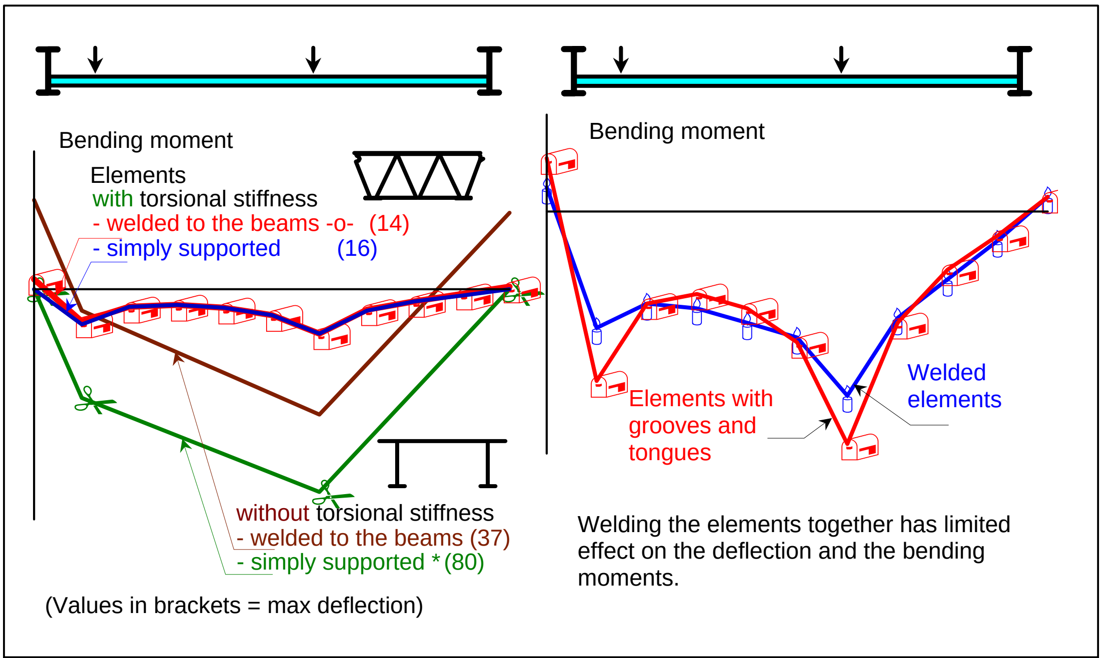
1. Closed elements with large torsional stiffness
2. Open profiles with small torsional stiffness
3. Closed profiles welded together in the top and bottom flange



TALAT Training in Aluminium Application Technologies | **Supporting beam and deck elements of a hoistable deck for ferries** | 2301.08.05

The following results can be drawn in the figure:

1) The bending moment in a deck of open profiles is large. It is almost the same as if a single element carries the whole load;
2) The bending moment is much less in hollow section deck elements. The load is spread out so that about five elements share the load;
3) Welding the elements together to a deck with almost the same bending stiffness in both transverse and longitudinal directions have only slight influence on the bending moment in the longitudinal direction. As the strength is reduced in the heat-affected zone around the weld, the strength of the deck will not necessarily increase by the welds.
4) Furthermore, the negative bending moment between the deck elements and the beams is very small when hollow section profiles are used. That is due to the spread of the load on many adjacent elements. In an open section deck the negative moments at the supports are almost the same as if the ends of the elements were completely fixed.



TALAT Training in Aluminium Application Technologies | Bending moment and deflection of an hoistable deck for ferries | 2301.08.06

8.03 Torsion without warping

The torsional resistance of a member with an open cross section is given by

$$T_{Rd} = 0{,}58\, W_{p,v} \frac{f_o}{\gamma_{M1}} \tag{8.1}$$

where $W_{p,v}$ = plastic torsion constant.

The torsion strength of a hollow cross section composed by flat part is

$$T_{Rd} = \rho_v\, 2\, A\, t_w \frac{f_o}{\gamma_{M1}} \tag{8.2}$$

where
A = area enclosed by the midline of the hollow section
t_w = thickness of designing flat part of the cross section, normally the thinnest part
ρ_v = reduction factor for shear buckling which is a function of λ_w

$$\lambda_w = 0{,}35 \frac{b_w}{t_w} \sqrt{\frac{f_o}{E}} \tag{8.3}$$

is determined for the considered cross section part with width-to-thickness b_w/t_w.

No special reduction factor for shear buckling by torsion of box beams is given in Eurocode 9. As the post-buckling range is less pronounced for thin-walled hollow sections in torsion than in shear, a somewhat lower value than for shear of webs is recommended in table 8.01.

Table 8.01. Reduction factor ρ_v with regard to shear buckling of hollow cross sections in torsion.

λ_w	ρ_v
$\lambda_w < 0{,}65$	0,67
$0{,}65 \leq \lambda_w \leq 2{,}37$	$0{,}435/\lambda_w$
$2{,}37 < \lambda_w$	$0{,}58/\lambda_w^{4/3}$

8.04 Torsion with warping

In the case of a member subjected to torsion and where warping is prevented, or, in the case where torsion is combined with bending and/or axial load, warping stresses are to be taken into consideration.

Normal stresses $\sigma_{E,sum}$ caused by normal force, bending moment and warping and shear stress $\tau_{E,sum}$ from St.Venant's shear and warping shear shall fulfill the expression

$$\sqrt{\sigma_{E,sum}^2 + 3\tau_{E,sum}} < 1{,}1\frac{f_o}{\gamma_{M1}} \qquad (8.4)$$

9 Axial force and bending moment

9.01 General

In this clause interaction expressions for checking members subjected to a combination of axial force and major axis and/or minor axis bending are discussed.

In general three checks are needed

- section check
- flexural buckling check
- lateral-torsional buckling check.

A section check is usually sufficient for beam-columns in tension and bending, unless the bending moment can cause lateral-torsional buckling. For compression and bending no special section check is needed, as it is included in the check for flexural buckling and lateral-torsional buckling in the interaction expressions.

When calculating the resistance N_{Rd}, $M_{y,Rd}$ and $M_{z,Rd}$ in the interaction expressions, due account of the presence of HAZ-softening from longitudinal welds should be taken as well as local buckling and yielding. The presence of localised HAZ-softening from transverse welds and the presence of holes is treated in 9.08 and 9.09 respectively.

Classification of cross sections for members with combined bending and axial force is made for the loading components separately. No classification is made for the combined state of stress.

This means that a cross section can belong to different classes for axial force, major axis bending and minor axis bending. The combined state of stress is allowed for in the interaction expressions. Therefore, the interaction expressions can be used for all classes of cross section. The influence of local buckling and yielding on the resistance for combined loading is taken care of by the resistance N_{Rd}, $M_{y,Rd}$ and $M_{z,Rd}$ in the denominators and the exponents which are functions of the slenderness of the cross section.

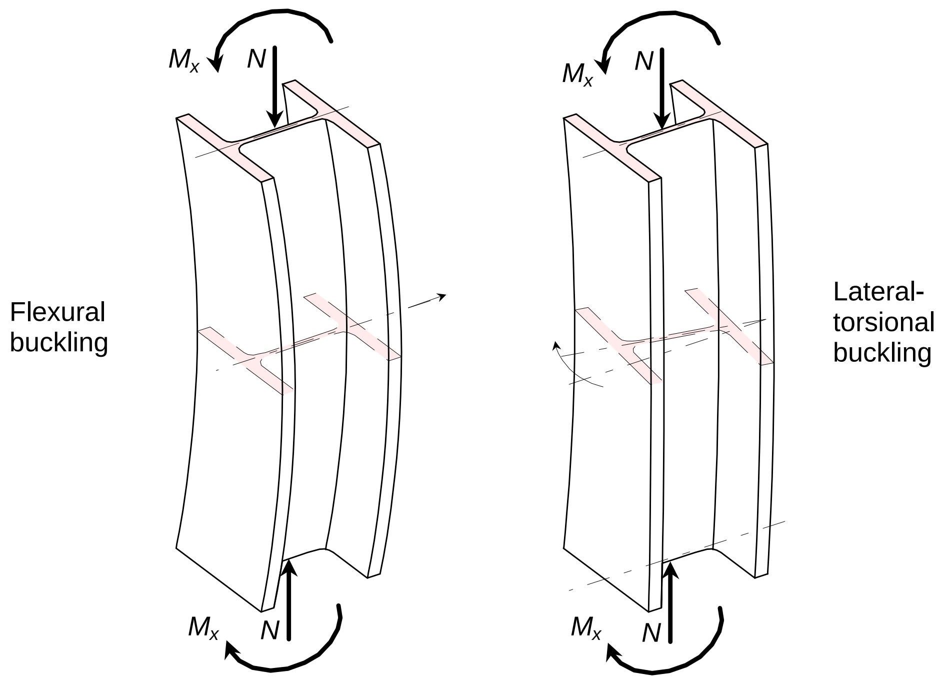
9.02 Bending and axial tension

A section check is usually sufficient for beam-columns in tension and bending, unless the bending moment can cause lateral-torsional buckling. Slender members shall therefore be checked for resistance to lateral-torsional buckling, treating the axial force and bending moment as vectorial effects.

Members subjected to combined bending and axial tension shall satisfy expressions for the strength of beam column segments in 9.04.

9.03 Bending and axial compression

A beam-column in compression and bending can fail by flexural buckling or by lateral-torsional buckling. The deformations corresponding to these buckling modes are shown in **Figure 2301.09.01**.



TALAT alu Training in Aluminium Application Technologies	Flexural buckling and lateral torsional buckling	2301.09.01

The resistance of aluminium beam-columns with regards to flexural buckling and lateral torsional buckling depends on a number of factors:

- geometrical, such as length of beam-column, shape of cross section and conditions at the beam-column ends,
- material related, such as stress-strain curve, residual stresses and heat affected zones,
- imperfections e.g. initial deflections.
- loading conditions.

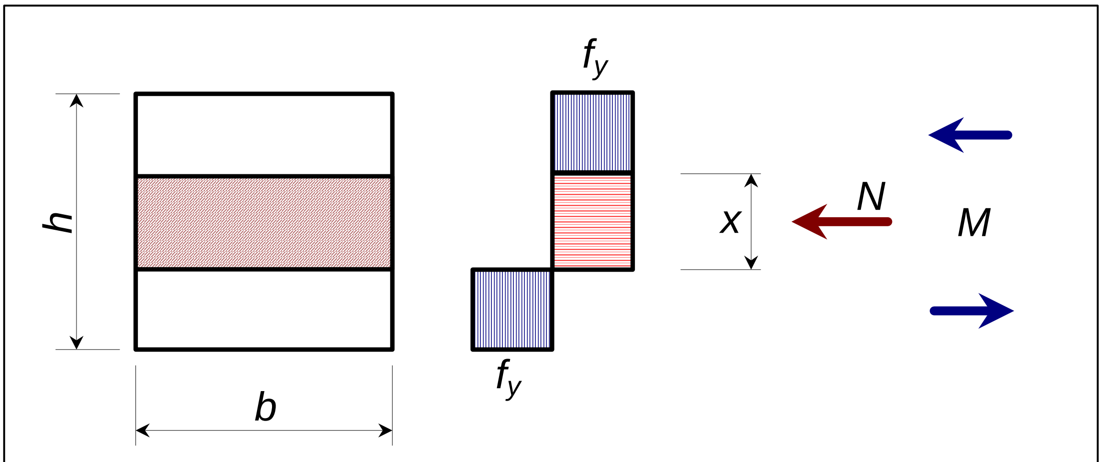
Typical for aluminium is the strain hardening condition. To illustrate the influence of the stress strain curve, in 9.04, beam-column segments of ideal elastic-plastic material and strain hardening material are compared. In 9.05 flexural buckling is treated, in 9.06 lateral-torsional buckling, in 9.07 thin-walled sections, in 9.08 the influence of transverse welds, in 9.09 influence of bolt holes or cut-outs and in 9.10 the influence of variation of the bending moment along the beam-column.

9.04 Strength of beam-column segments.

In the following the *strength* and *behavior* of beam-column *segments* subjected to compression combined with biaxial bending are presented. As the name implies, here we are only concerned with *short* beam-columns for which the effect of lateral deflections on the magnitudes of bending moments is negligible. As a result, the maximum strength occurs when the entire cross section is fully plastic or yielded in the case of elastic-plastic material as mild steel or when the maximum strain (or stress) attains some prescribed value in the case of hardening material such as aluminium. This is known as stress problem of first order. Material yielding or failure is the primary cause of the strength limit of the member. Method of analysis in predicting this limiting strength is presented here as a background to Eurocode 9.

Rectangular section - plastic theory

If the slenderness *b/t* of the cross sectional parts is small and the strain at rupture large, the whole cross section may yield. For a solid, rectangular section of ideal elastic-plastic material, the relation between axial force and bending moments for rectangular stress distribution is easily derived.



TALAT Training in Aluminium Application Technologies	Rectangular section in bending and compression	2301.09.02

The central part *xb* of the rectangular cross section in **Figure 2301.09.02** is assumed to carry the axial force *N*

$$N = f_y \, b \, x = N_{pl} \frac{x}{h} \qquad (9.01)$$

where $N_{pl} = f_y \, bh$. (9.02)

The bending moment, *M*, is carried by two stress blocks, each having a depth equal to $(h - x)/2$ which gives

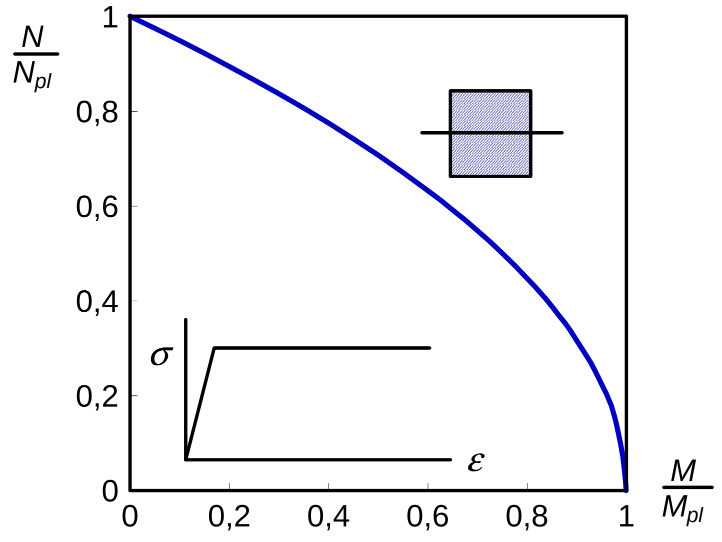
$$M = f_y \frac{h - x}{2} b \frac{h + x}{2} = M_{pl} \left(1 - \frac{x^2}{h^2} \right) \quad (9.03)$$

$$\text{where } M_{pl} = \frac{f_y \, b h^2}{4} \quad (9.04)$$

If *x/h* according to (9.01) is inserted into (9.03) the following interaction formula is obtained:

$$\left(\frac{N}{N_{pl}} \right)^2 + \frac{M}{M_{pl}} = 1 \quad (9.05)$$

In this equation, the first term, containing the axial force, is squared and the second term, containing bending moment, has no exponent (exponent = 1). The equation represents a parabola as shown in **Figure 2301.09.03**.



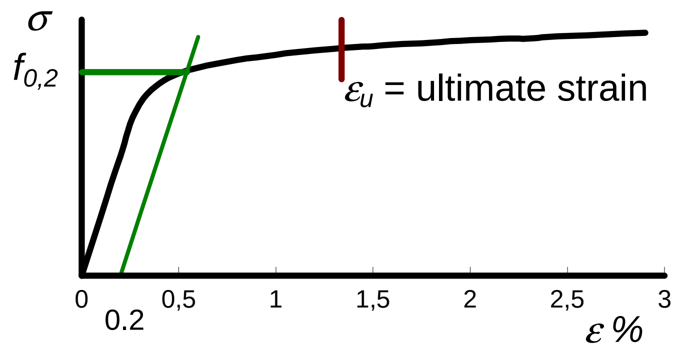
TALAT Training in Aluminium Application Technologies	Interaction diagram for a rectangular beam-column segment of ideal elastic-plastic material	2301.09.03

Rectangular section - strain hardening material

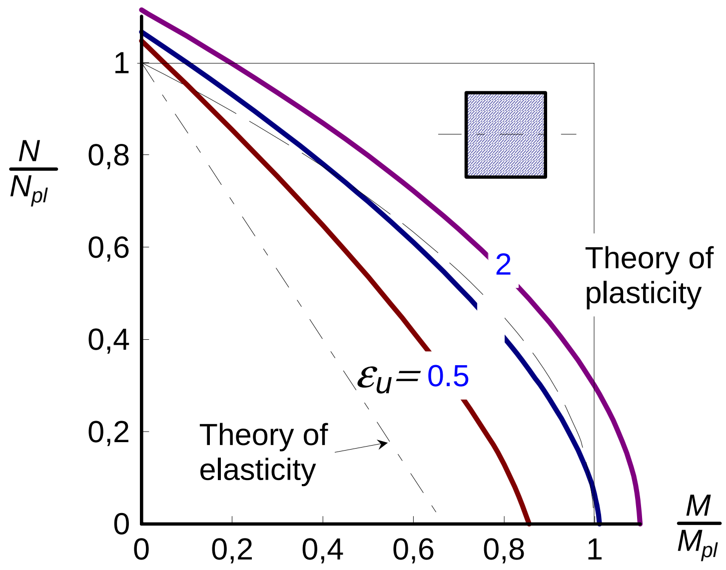
Similar curves can be derived for aluminium beam-columns. The bending moment is given by integrating the stresses over the cross section assuming linear strain distribution. As the strain ε in the Ramberg-Osgood expression is a function of the stress σ, then numerical integration is used dividing the cross section into small elements. The shape of the curve depends on the stress-strain relationship and the limiting strain ε_u. Curves are shown in **Figure 2301.09.05** for aluminium having a strain hardening parameter n = 15, in the Ramberg-Osgood expression:

$$\varepsilon = \frac{\sigma}{E} + 0{,}002\left(\frac{\sigma}{f_{0,2}}\right)^{n} \qquad (9.06)$$

The stress-strain relationship of this material is shown in **Figure 2301.09.04**.



TALAT — Training in Aluminium Application Technologies | Stress-strain relationship according to Ramberg-Osgood | 2301.09.04

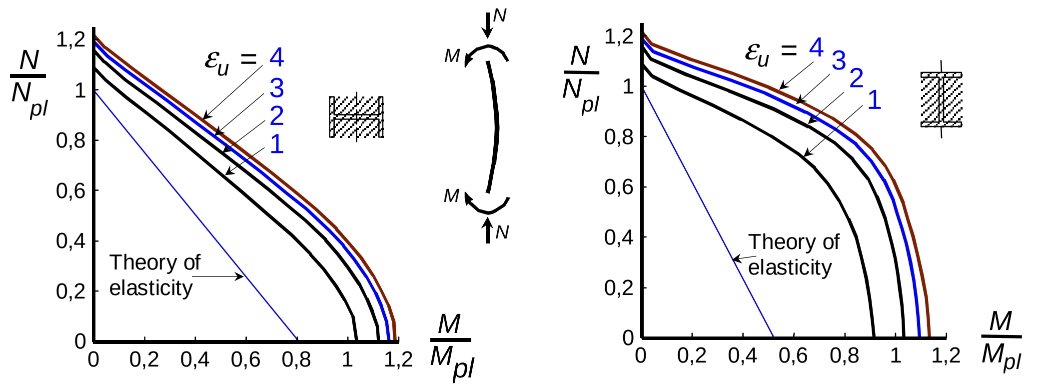


TALAT — Training in Aluminium Application Technologies | Interaction curves for a rectangular aluminium beam-column | 2301.09.05

If the limiting compressive strain is ε_u= 0,01, which is about twice the strain corresponding to the proof stress $f_{0,2}$, then the curve is very close to that for ideal elastic-plastic material (dashed curve).

I-, H- and T-section - strain hardening material

Interaction curves are given in Figures 9.06 and 07 for aluminium beam-column segments with other types of symmetrical cross sections. For minor axis bending of H-sections, the curves are strongly convex, cf. **Figure 2301.09.06**.



H beam-column, *y*-axis bending   H beam-column, *z*-axis bending

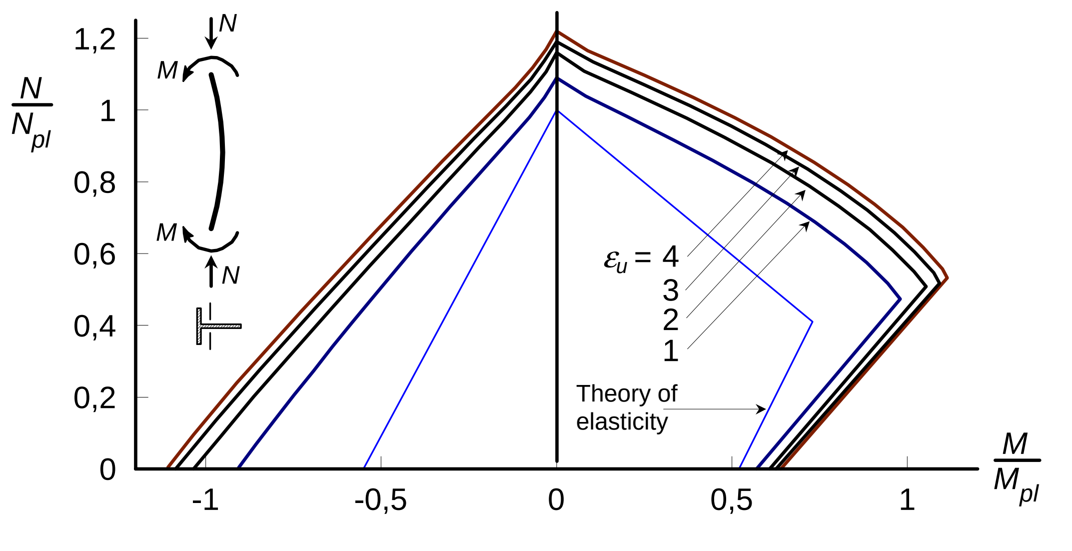
TALAT Training in Aluminium Application Technologies	Interaction curves for H beam-column segment	2301.09.06

For unsymmetrical sections the shape of the interaction curve depends on the direction of the bending moment, cf. Figures 9.07.

The thin lines in Figures 9.06 and 07 show the resistance according to the theory of elasticity. As illustrated, the plastic resistance is far greater than that according to the theory of elasticity, even though the maximum strain, ε_u, is only 1%. The *shape* of the curves is mostly independent of the magnitude of the maximum strain, but the level of the resistance is increased with increasing strain.

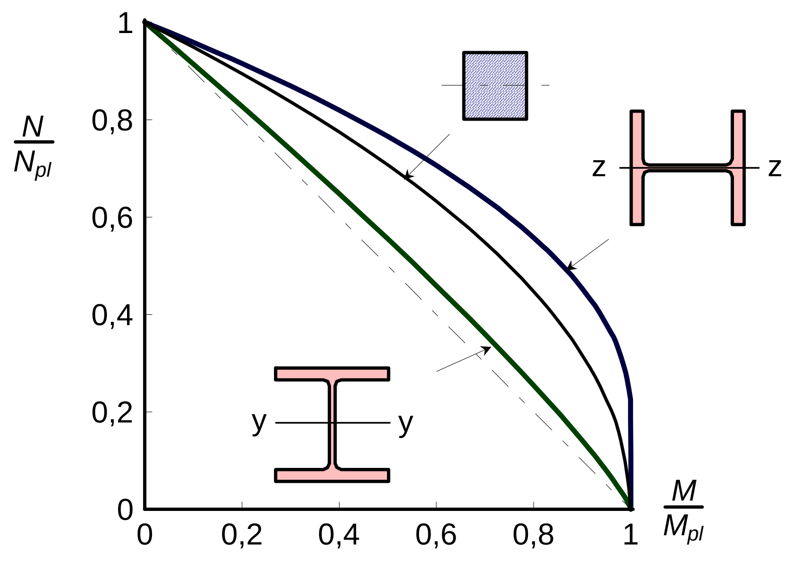
The straight lines starting from the positive M/M_{pl} - axis in Figures 9.07 are valid when the strain on the *tension* side, is equal to the maximum strain ε_u.

As noted above, the curves for ε_u = 0,01 are very close to those for ideal elastic-plastic material. Then, the following curves and expressions for elastic-plastic material are approximately valid for aluminium.



TALAT Training in Aluminium Application Technologies	Interaction curves for a T beam-column segment	2301.09.07

For other types of cross sections besides rectangular, it is not possible to obtain a simple relationship between N/N_{pl} and M/M_{pl} such as that in equation (9.05), even for ideal elastic-plastic material. For an H beam-column subjected to *z*-axis bending, as shown in **Figure 2301.09.08**, the interaction curve lies above that for a rectangular section. The reason for this is that the web can carry the axial force and the flanges carry the bending moment.

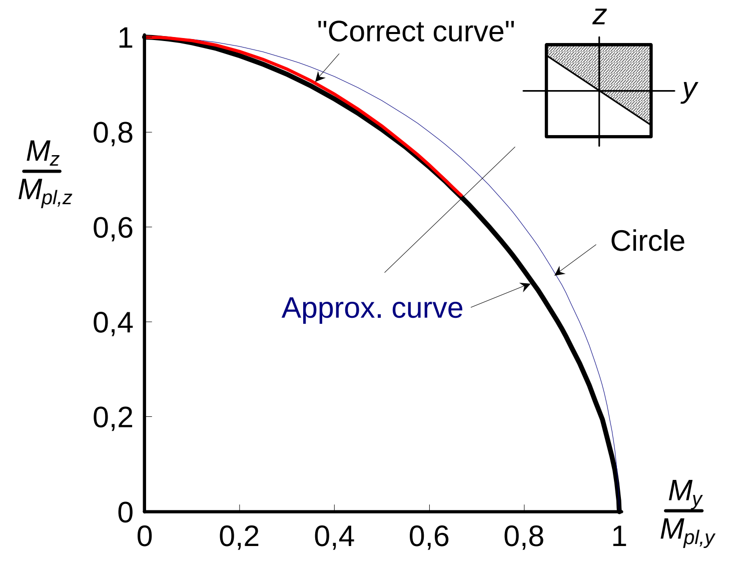


TALAT Training in Aluminium Application Technologies	Interaction curves for axial force and y- or z-axis bending	2301.09.08

For *y*-axis bending, the interaction curve is close to a straight line between the coordinates (0;1) and (1;0). The contribution from the web is the reason for deviation from a straight line.

Biaxial bending of rectangular section

For a rectangular section, a simple interaction formula can be derived for zero axial force, noting that the area of the cross section is divided into two equal parts by the inclined line, cf. **Figure 2301.09.09**.



TALAT Training in Aluminium Application Technologies	Bi-axial bending of beam with rectangular cross section. *N* = 0	2301.09.09

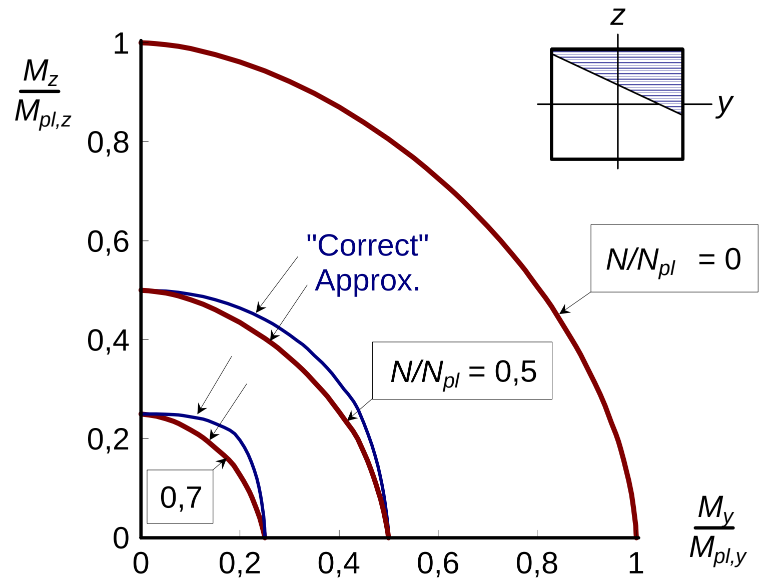
The correct expression for the upper part ($M_z/M_{pl,z} > 2/3$) is:

$$\frac{M_z}{M_{pl,z}} + \frac{3}{4}\left[\frac{M_y}{M_{pl,y}}\right]^2 \leq 1{,}00 \qquad (9.07)$$

For the other half of the curve, index *y* and *z* change place. The following expression is an approximation for all values of $M_z/M_{pl,z}$. The difference in result between the expressions is less than 1%.

$$\left[\frac{M_y}{M_{pl,y}}\right]^{1{,}7} + \left[\frac{M_z}{M_{pl,z}}\right]^{1{,}7} \leq 1{,}00 \qquad (9.08)$$

When an axial force is added, the relationship illustrated by the “correct” curves in **Figure 2301.09.10** is obtained. The curves are convex and the convexity increases as the axial force is increased. The following expression corresponds to the “approx.” curves which provide results on the safe side except when M_y =0 or M_z =0. Then the formula transforms into the correct expression (9.05), which corresponds to the parabola in **Figure 2301.09.03**.

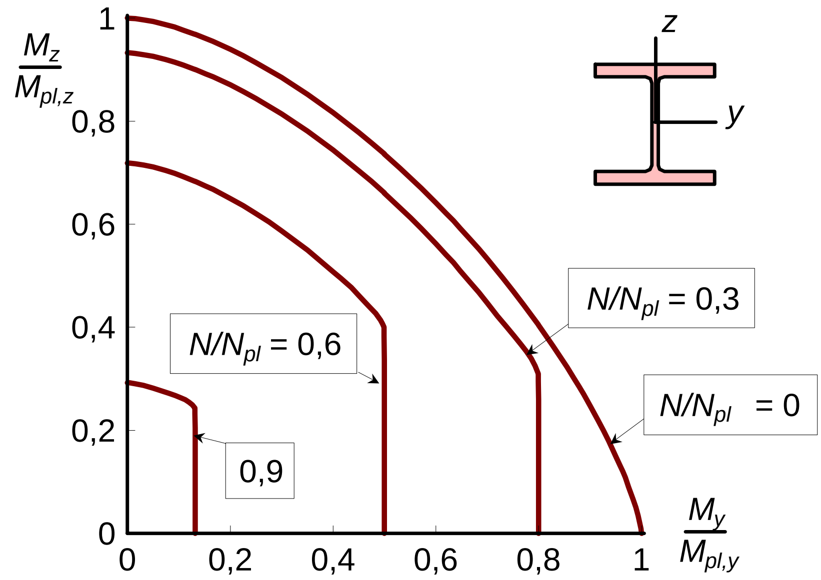


TALAT Training in Aluminium Application Technologies | Interaction curves for rectangular beam-columns subjected to axial force and bi-axial | 2301.09.10

$$\left[\frac{N}{N_{pl}}\right]^2 + \left(\left[\frac{M_y}{M_{pl,y}}\right]^{1,7} + \left[\frac{M_z}{M_{pl,z}}\right]^{1,7}\right)^{0,6} = 1{,}00 \qquad (9.09)$$

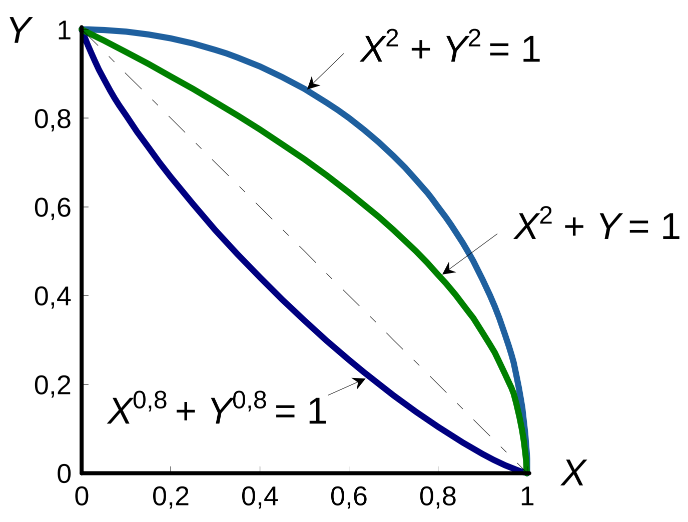
Biaxial bending of I- and H-section

For H-beam-columns the convexity is even greater, cf. **Figure 2301.09.11**. Calculations have shown that the curves in the figure represent H-beam-columns with various sectional dimensions.



TALAT Training in Aluminium Application Technologies	Interaction curves for H beam-column segment in bi-axial bending and compression	2301.09.11

By choosing suitable exponents for the quotients N/N_{pl} and M/M_{pl} it is possible to obtain curves that correlate well with the "real" curves, cf. **Figure 2301.09.12**. If the exponents exceed the value one, the curves are convex. If the exponents are below one, the curves are concave.



TALAT Training in Aluminium Application Technologies	Influence of exponents on the form of interaction curves	2301.09.12

An asymmetrical curve is obtained when one exponent is larger than the other, as illustrated in **Figure 2301.09.12**. This method was used for choosing exponents for interaction formulae in Eurocode 9. The resulting expressions are shown in **Figure 2301.09.13**. The exponent η_0 for $M_{y,Ed}/M_{y,Rd}$ depends on the shape factor a_z and the exponent γ_0 for $M_{z,Ed}/M_{z,Rd}$ depends on α_y. These factors are determined as $a_z = W_{z,pl}/W_{z,el}$ and $a_y = W_{y,pl}/W_{y,el}$ for class 1 and 2 cross sections, and less for class 3 and 4 cross sections. This means that the exponents are smaller, and consequently the curves are less convex for slender cross sections.

The first formula (9.10) in **Figure 2301.09.13** corresponds to the vertical parts of the interaction curves in **Figure 2301.09.14.** The second formula corresponds to the curved parts, and is valid only for values of $M_{z,E}/M_{z,R}$ larger than 0,2 - 0,35 depending on the axial force.

$$\left(\frac{N_E}{N_R}\right)^{\xi_0} + \frac{M_{y,E}}{M_{y,R}} \leq 1{,}00 \qquad (9.10)$$

and

$$\left(\frac{N_E}{N_R}\right)^{\eta_0} + \left(\frac{M_{y,E}}{M_{y,R}}\right)^{\gamma_0} + \left(\frac{M_{z,E}}{M_{z,R}}\right)^{\xi_0} \leq 1{,}00 \qquad (9.11)$$

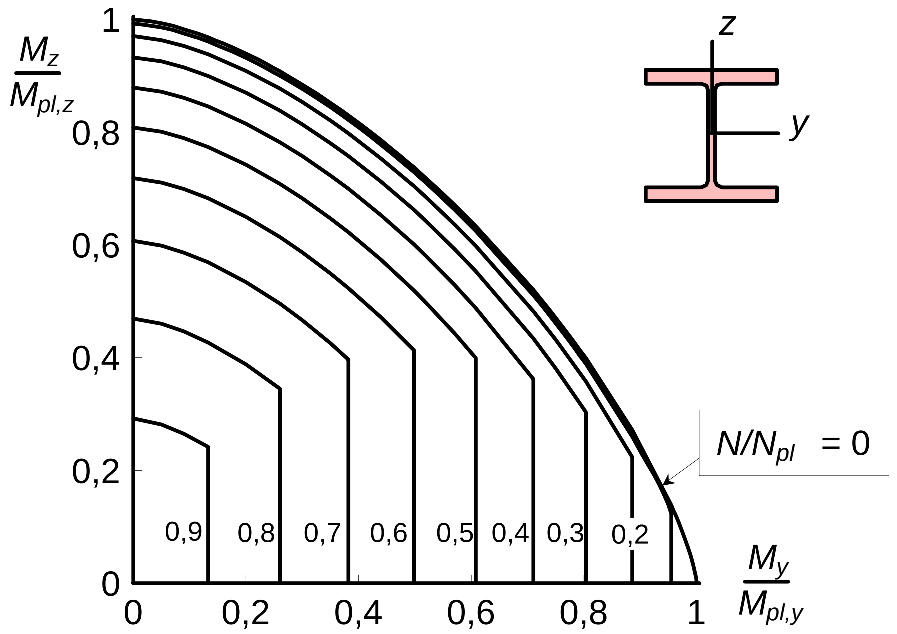
where the exponents η_0, γ_0 and ξ_0 are

$$\eta_0 = \alpha_z^2 \alpha_y^2 \text{ but } \geq 1 \text{ and } \leq 2 \qquad (9.12)$$

$$\gamma_0 = \alpha_z^2 \text{ but } \geq 1 \text{ and } \leq 1{,}56 \qquad (9.13)$$

$$\xi_0 = \alpha_y^2 \text{ but } \geq 1 \qquad (9.14)$$

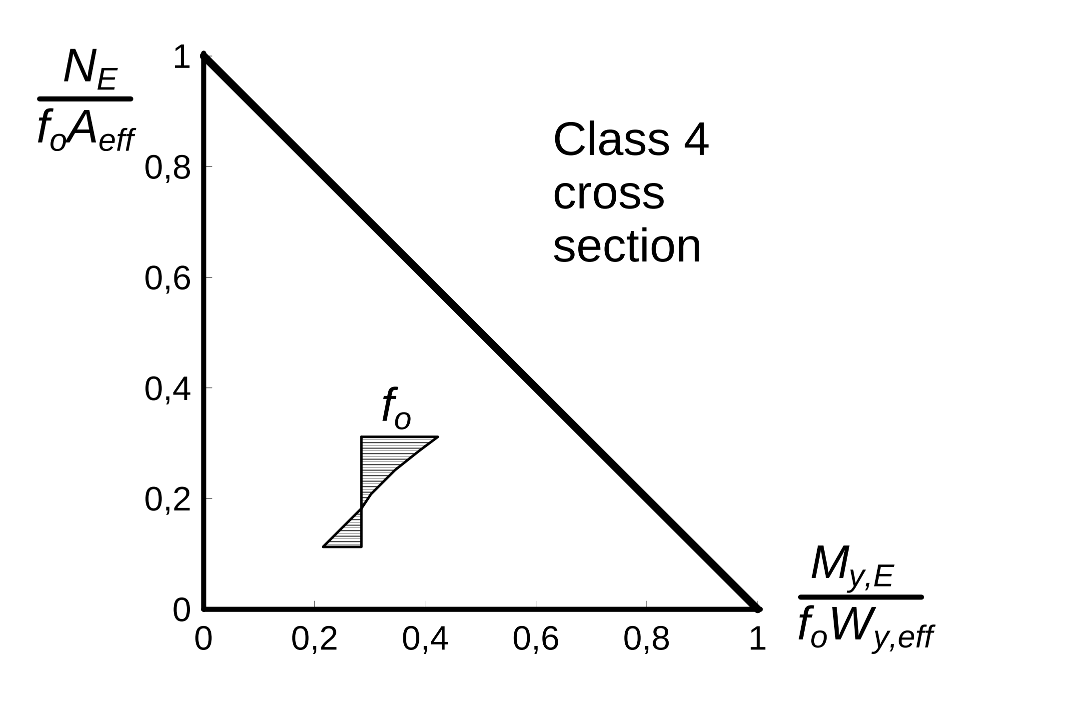
TALAT Training in Aluminium Application Technologies	**Interaction formulae for I-beam-column segment resistance**	2301.09.13



TALAT Training in Aluminium Application Technologies | Interaction curves for I beam-column segment | 2301.09.14

The choice of exponents as functions of the shape factors allows the possibility of taking local buckling into consideration. If the compression flange of a beam is so slender that local buckling occurs just as the stress reaches the yield stress, then the section will not yield and the interaction curve consists of a straight line between the coordinates (0;1) and (1;0), cf. **Figure 2301.09.15**.

One condition for the convex interaction curves is that the slenderness of the sectional parts is so small that they can yield in compression. If the slenderness is large, then the ultimate resistance is reduced due to local buckling (class 4 cross section) and the shape factor is calculated as $a = W_{eff}/W_{el}$ where W_{eff} is the section modulus of the reduced, effective section allowing for local buckling. The shape factors are less than one for a class 4 cross section, but the exponents in the interaction formula should not be less then one.



TALAT Training in Aluminium Application Technologies | Interaction relationship for thin-walled cross section | 2301.09.15

No yielding occur:

$$\sigma = \frac{N_E}{A_{eff}} + \frac{M_E}{W_{eff}} = f_o \tag{9.15}$$

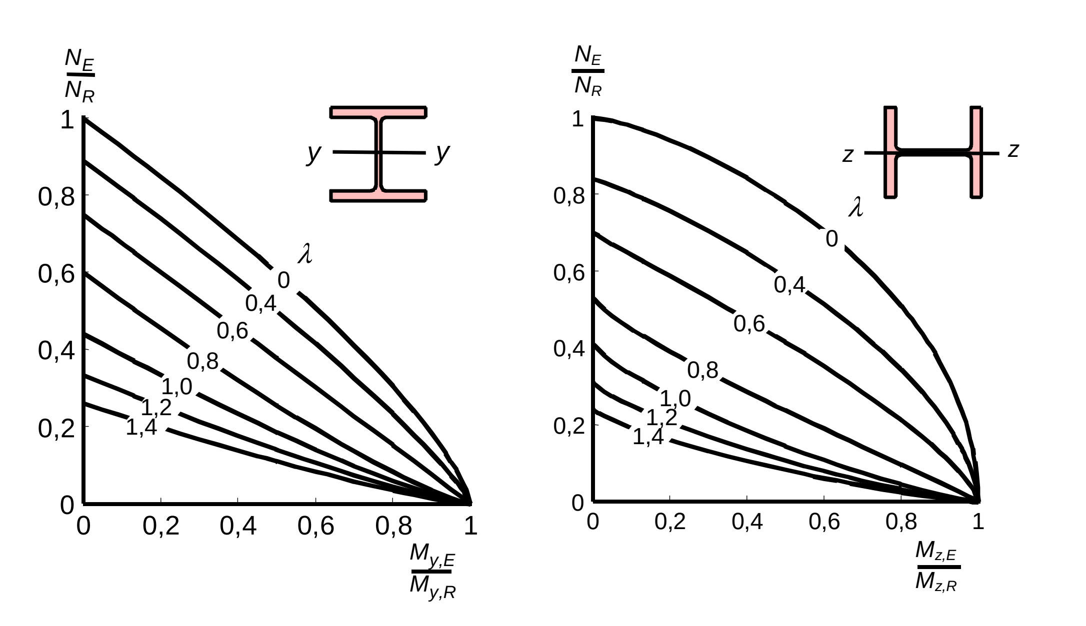
Then:

$$\frac{N_E}{A_{eff} \quad f_o} + \frac{M_E}{W_{eff} \quad f_o} = 1 \tag{9.16}$$

9.05 Flexural buckling

Second order moments result in change of the form of the interaction curves, see interaction diagrams in **Figure 2301.09.16**. (In the diagrams index *d* for *design value* is omitted). For beam-columns with very small slenderness parameter

$$\lambda = \frac{KL}{i} \frac{1}{\pi} \sqrt{\frac{f_o}{E}} \tag{9.17}$$



TALAT Training in Aluminium Application Technologies	Interaction diagrams for y-axis and z-axis bending of H beam-columns	2301.09.16

the second order moment is small and the curves therefore do not differ from the curves for a beam-column segment. For increasing slenderness ratio *KL/i* the curves start to deflect downward, resulting in concave curves for large slenderness ratio. This behavior is achieved by, for larger slenderness ratios, reducing the exponents in the interaction formulae for beam-column segments. This, in turn, is made by multiplying the exponents by the reduction factor χ for flexural buckling of the axially compressed member. The resulting expressions are:

∃ *y-axis bending of an I-beam-column*

$$\left(\frac{N_{Ed}}{\chi_y N_{Rd}}\right)^{\xi_{yc}} + \frac{M_{y,Ed}}{M_{y,Rd}} \leq 1{,}00 \qquad (9.18)$$

∃ *z-axis bending of an I-beam-column*

$$\left(\frac{N_{Ed}}{\chi_z \, N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{z,Ed}}{M_{z,Rd}}\right)^{\xi_{zc}} \leq 1{,}00 \qquad (9.19)$$

where the exponents are:

$$\eta_c = \eta_0 \chi_z \quad \text{but } \eta_c \geq 0{,}8 \qquad (9.20)$$

$$\xi_{yc} = \xi_0 \chi_y \quad \text{but } \xi_{yc} \geq 0{,}8 \qquad (9.21)$$

$$\xi_{zc} = \xi_0 \chi_z \quad \text{but } \xi_{zc} \geq 0{,}8 \qquad (9.22)$$

with η_0 and ξ_0 according to **Figure 2301.09.13**.

Note that these expressions are identical to the expressions in **Figure 2301.09.13** except that χ_y is added in the denominator in the first term in 9.18 and χ_z in 9.19. This mean that the denominator is equal to the flexural buckling resistance for the axially compressed column in the *x-z* and the *x-y* plane respectively. Furthermore, the $M_{y,Ed}/M_{y,Rd}$ term in the second interaction formula in **Figure 2301.09.13** is deleted as flexural buckling in the *x-y* plane is considered only.

∃ *Solid cross sections*

As bending and compression of a rectangular beam-column is very similar as *y*-axis bending of an I-beam-column, expression 9.19 is used with the exponents taken as:

$$\eta_c = 2\chi \quad \text{but } \eta_c \geq 0{,}8 \qquad (9.23)$$

$$\xi_c = 1{,}56\ \chi \quad \text{but } \xi_c \geq 0{,}8 \qquad (9.24)$$

∃ *Hollow cross sections and pipes*

The expression 5.25 is used with ψ_c taken as $\chi_y\psi$ or $\chi_z\psi$ depending on direction of buckling, but $\psi_c \leq 0{,}8$. χ_{min} is the lesser of χ_y and χ_z.

$$\left(\frac{N_{Ed}}{\chi_{min} N_{Rd}}\right)^{\psi_c} + \left[\left(\frac{M_{y,Ed}}{M_{y,Rd}}\right)^{1,7} + \left(\frac{M_{z,Ed}}{M_{z,Rd}}\right)^{1,7}\right]^{0,6} \leq 1{,}00 \qquad (5.25)$$

For hollow sections, ψ = 1,3. Note that this expression gives almost the same result as expression 9.18 for y-axis bending of an I-beam-column if $M_{y,Ed} = 0$ (or $M_{z,Ed} = 0$) as the resulting exponent is 1,7 times 0,6 which is close to 1,0 and ξ_0 is close to 1,3.

∃ *Other cross sections*

Expression 5.18 may be used for symmetrical and bi-symmetrical cross sections, bending about either axis, for *z*-axis bending replacing ξ_{yc}, $M_{y,Ed}$, $M_{y,Rd}$ and χ_y with ξ_{zc}, $M_{z,Ed}$, $M_{z,Rd}$ and χ_z.

The notations in expressions 5.18, 5.19 and expression 5.25 are (In the diagrams the index *d* for *design value* is omitted):

N_{Ed} = axial compressive force

N_{Rd} = Af_o/γ_{M1} (or $A_{ef}f_o/\gamma_{M1}$ for class 4 cross sections)

χ_y = reduction factor for buckling in the *z-x* plane

χ_z = reduction factor for buckling in the *y-x* plane

$M_{y,Ed}$,$M_{z,Ed}$ = bending moment about the *y*- and *z*-axis. The moments are calculated according to first order theory

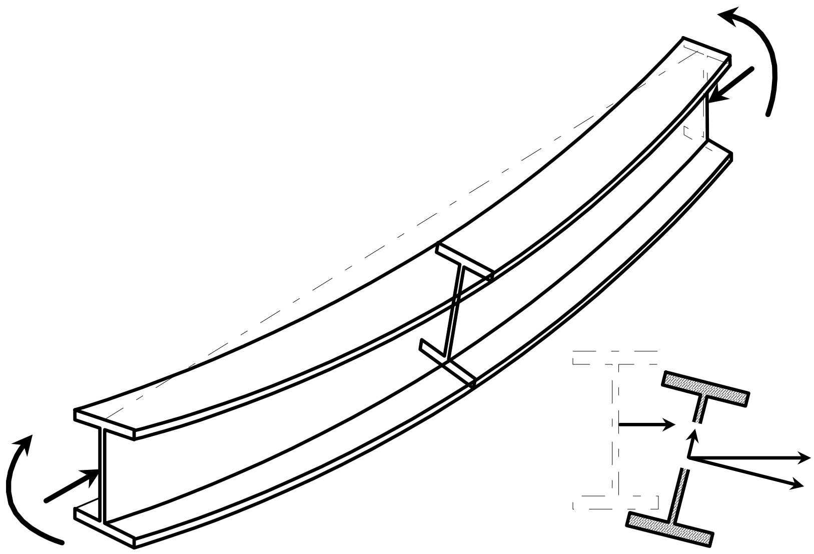
$M_{y,Rd}$ = $a_y W_y f_o/\gamma_{M1}$ bending moment resistance about the *y*-axis

$M_{z,Rd}$ = $a_z W_z f_o/\gamma_{M1}$ bending moment resistance about the *z*-axis

a_y , a_z = shape factor but a_z should not be taken larger than 1,25

9.06 Lateral-torsional buckling

When a beam is bent in its stiffer principal plane, it usually deflects in that plane. However, if the beam does not have sufficient lateral stiffness or lateral support to insure that it is so, then the beam may buckle out of the plane of loading. The load at which this buckling occurs may be substantially less than the in-plane load carrying capacity of the beam. While short beams can reach the full plastic moment M_{pl} more slender beams may buckle at moments M_{cr} which are significantly less than M_{pl}. For an idealized perfect straight elastic beam, there are no out-of-plane displacements until the applied moment reaches the critical value M_{cr}, when the beam buckles by deflecting laterally and twisting. Thus lateral buckling involves lateral bending and axial torsion. These two actions are interdependent, and when the beam deflects laterally, the applied moment exerts a component torque about the deflected longitudinal axis which causes the beam to twist, while twisting of the beam causes the applied moment to exert a component lateral bending moment which causes the beam to deflect. This behavior, which is important for unrestrained I-beams whose resistance to lateral bending and torsion are law, is called elastic flexural-torsional buckling.



TALAT Training in Aluminium Application Technologies	Lateral-torsional buckling	2301.09.17

A beam-column which is bent in its stiffer principal plane and compressed, see **Figure 2301.09.17**, may also fail predominantly by buckling out of the plane of loading. This behaviour is closely related to the flexural-torsional buckling of beams. However, the risk of lateral buckling is even larger as the axial compression force tend to cause weak axis flexural buckling by itself.

As for flexural buckling the following expression for the resistance of a beam-column segment is used as a starting point.

$$\left(\frac{N_E}{N_R}\right)^{\eta_0} + \left(\frac{M_{y,E}}{M_{y,R}}\right)^{\gamma_0} + \left(\frac{M_{z,E}}{M_{z,R}}\right)^{\xi_0} \le 1{,}00 \qquad (9.26)$$

The effect of second order moments are introduced by:

- adding the reduction factor χ_z for *z*-axis flexural buckling of the axially compressed member in the first denominator.
- adding the reduction factor χ_{LT} for lateral-torsional buckling for the bending moment $M_{y,E}$ in the denominator in the second term.
- multiplying the exponents η_0 and ξ_0 with the reduction factor χ_z for *z*-axis flexural buckling of the axially compressed member.

Note however, that the exponent γ_0 is not multiplied with the reduction factor χ_z as buckling take place by out-of plane deflection and twisting. The second order moment is predominantly a M_z bending moment as flexure occur in the *x*-*y* plane.

The resulting expression for beam-columns with I shaped and similar cross sections is:

$$\left(\frac{N_{Ed}}{\chi_z N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{y,Ed}}{\chi_{LT}\; M_{y,Rd}}\right)^{\gamma_c} + \left(\frac{M_{z,Ed}}{M_{z,Rd}}\right)^{\xi_{zc}} \le 1{,}00 \qquad (5.27)$$

where:

N_{Ed} = axial force

$M_{y,Ed}$ = bending moments about the *y*-axis. In the case of beam-columns with hinged ends and in the case of members in non-sway frames, $M_{y,Ed}$ is moment of the *first order*. For members in frames free to sway, $M_{y,Ed}$ is bending moment according to *second order* theory.

$M_{z,Ed}$ = bending moments about the *z*-axis. $M_{z,Ed}$ is bending moment according to *first order* theory

N_{Rd} = Af_o/γ_{M1} or $A_{ef} f_o/\gamma_{M1}$ for class 4 cross sections.

χ_z = reduction factor for buckling when one or both flanges deflects laterally (buckling in the *y*-*z* plane or lateral-torsional buckling)

$M_{y,Rd} = a_y W_y f_o/\gamma_{M1}$ = the bending moment resistance for *y*-axis bending

χ_{LT} = reduction factor for lateral-torsional buckling

$M_{z,Rd} = a_z\, W_z\, f_o/\gamma_{M1}$ = the bending moment resistance for z-axis bending

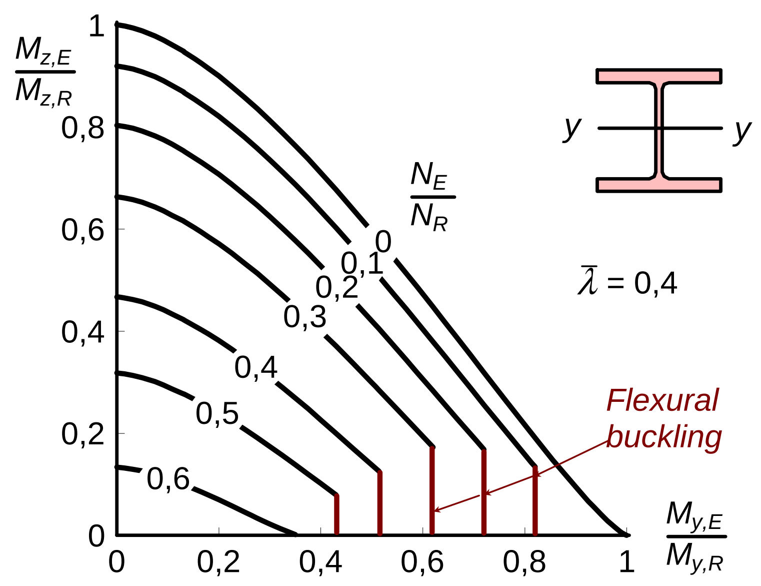
η_c = 0,8 or alternatively $\eta_0 \chi_z$ but $\eta_c \exists$ 0,8

γ_c = γ_0

ξ_{zc} = 0,8 or alternatively $\xi_0\, \chi_z$ but $\xi_{zc} \exists$ 0,8

η_0, γ_0 and ξ_0 are defined according to the expression in **Figure 2301.09.13**.

The expression for flexural buckling, see expression 9.19, must also be checked. This expression is governing for small z-axis bending moment. See examples of interaction curves in **Figure 2301.09.18** where the vertical straight-line parts of the interaction curves correspond to flexural buckling. The curves in **Figure 2301.09.18** are valid for a rather small slenderness parameter $\overline{\lambda}_y = 0{,}4$. In slender beam-columns, flexural buckling is more seldom governing.



TALAT Training in Aluminium Application Technologies

Example of interaction curves for lateral-torsional buckling

2301.09.18

Note that $M_{y,Ed}$ is the *first order* bending moment for beam-columns with hinged ends and for members in non-sway frames, but *second order* bending moment for members in frames free to sway. In most cases the buckling length for y-axis buckling is different from that for z-axis buckling. The buckling length for z-axis buckling is usually supposed to be the actual member length, whereas the y-axis buckling length usually is larger in sway frames and less in braced frames.

9.07 Thin walled cross sections

The choice of exponents as functions of the shape factors allows the possibility of taken local buckling into consideration, without changing the interaction formulae. If the most compressed flange is so slender that local buckling occurs, then that section will not yield and the interaction curve is close to a straight line. This means that the exponents should be close to 1 for a beam-column segment and close to 0,8 for flexural and lateral-torsional buckling.

9.08 Transverse welds

The influence of transverse welds on the buckling resistance depends, to some extent, on the location of the welds. If a weld is close to the mid-section of the beam-column, the buckling strength is about the same as if the hole column were made of heat-affected material. If the transverse welds are close to pinned ends, they do not reduce the buckling strength, but the squash load of the beam-column segment at the ends. This means that, if the slenderness of the column is large and consequently the buckling resistance is small compared to the squash load, then the transverse welds at the ends do not influence the resistance.

In Eurocode 9 the coefficient ω_0 is introduced in the interaction formulae for beam-column segments to take the influence of transverse welds into consideration. Furthermore, in the interaction formulae for flexural buckling and lateral-torsional buckling the coefficients ω_x and ω_{xLT} are introduced as follows

Flexural buckling, y-axis bending:

$$\left(\frac{N_{Ed}}{\chi_y\,\omega_x\,N_{Rd}}\right)^{\xi_{yc}}+\frac{M_{y,Ed}}{\omega_o\,M_{y,Rd}}\le 1{,}00 \qquad (9.28)$$

Flexural buckling, z-axis bending:

$$\left(\frac{N_{Ed}}{\chi_z\,\omega_x\,N_{Rd}}\right)^{\eta_c}+\left(\frac{M_{z,Ed}}{\omega_o\,M_{z.Rd}}\right)^{\xi_{zc}}\le 1{,}00 \qquad (9.29)$$

Lateral-torsional buckling:

$$\left(\frac{N_{Ed}}{\chi_z\,\omega_x\,N_{Rd}}\right)^{\eta_c}+\left(\frac{M_{y,Ed}}{\chi_{LT}\,\omega_{xLT}\,M_{y,Rd}}\right)^{\gamma_c}+\left(\frac{M_{z,Ed}}{\omega_o\,M_{z,Rd}}\right)^{\xi_{zc}}\le 1{,}00 \qquad (9.30)$$

Conservatively, the following value can be used:

$$\omega_o=\omega_x=\omega_{xLT}=\frac{\rho_{haz}\,f_a/\gamma_{M2}}{f_o/\gamma_{M1}} \quad \text{but} \le 1{,}00 \qquad (9.31)$$

where ρ_{haz} is the reduction factor for the heat affected material according to 9.4.2.

However, when HAZ softening occurs close to the ends of the bay, or close to points of contra flexure only, ω_x and ω_{xLT} may be increased in considering flexural and lateral-torsional buckling, provided that such softening does not extend a distance along the member greater than the least width of the section.

$$\omega_x = \frac{\omega_o}{\chi + (1 - \chi)\sin\frac{\pi x_s}{l_c}} \tag{9.32}$$

$$\omega_{xLT} = \frac{\omega_o}{\chi_{LT} + (1 - \chi_{LT})\sin\frac{\pi x_s}{l_c}} \tag{9.33}$$

where

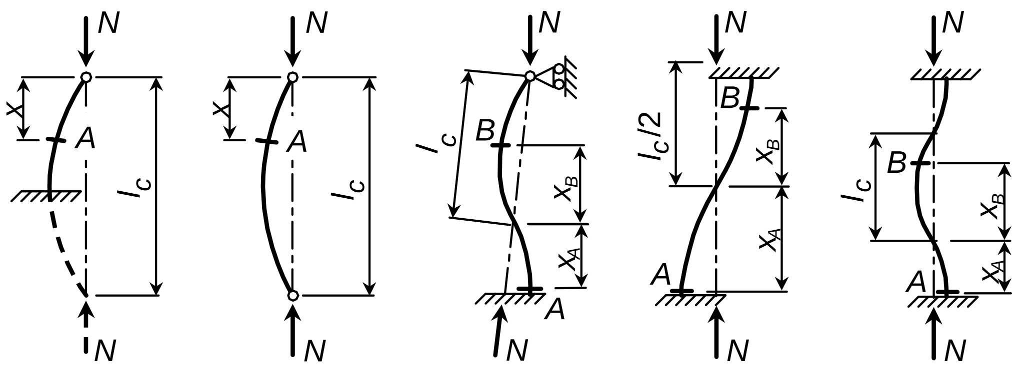
$$\omega_o = \frac{\rho_{haz} f_a / \gamma_{M2}}{f_o / \gamma_{M1}} \quad \text{but } \omega_o \le 1{,}00 \tag{9.34}$$

χ = χ_y or χ_z depending on buckling direction

χ_{LT} = reduction factor for lateral-torsional buckling of the beam-column

x_s = distance from the localized weld to a pinned support or point of contra flexure for the deflection curve for elastic buckling of axial force only, compare **Figure 2301.09.19**

l_c = effective buckling length.



TALAT Training in Aluminium Application Technologies

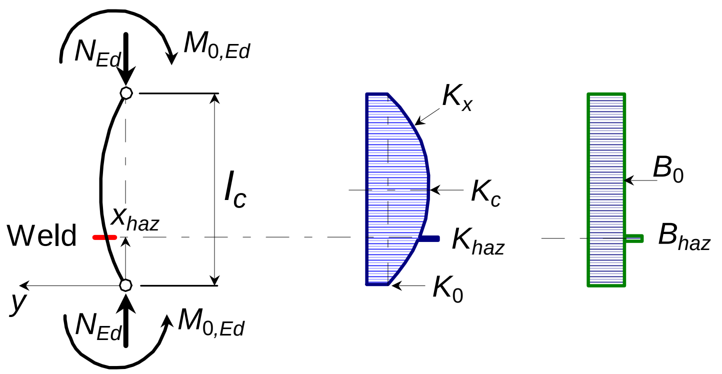
Effective buckling length l_c and definition of x

2301.09.19

In **Figure 2301.09.19** A and B mark examples of studied sections on the distance x_A and x_B from a pinned end or point of contra-flexure. See table 5.07 for values of buckling length $l_c = KL$.

The design procedure is illustrated in **Figure 2301.09.20** where K is short for the first quotient in expression 9.28 and B is short for the second quotient. K_x varies along the beam-column. At the pinned ends $K_x = K_0 = N_{Ed} / (Af_o/\gamma_{M1})$ corresponding to a Astress resistance ratio”. At the mid-section $K_x = K_c = K_0 / \chi$ corresponding to a Abuckling resistance ratio”. In-between K_x varies as a sine curve, see **Figure 2301.09.20** and the expression below the figure.

In the section of the transverse weld K_x and B is increased by $1/\omega_0$. In this example two sections need to be checked; the mid-section and the section with the weld. If the applied bending moment varies along the beam-column, then in principle all sections need to be checked, to find the largest value of $K^{\xi} + B$. In most cases it is possible to identify intuitively which section is governing.



$$K_0 = \frac{N_{Ed}}{N_{Rd}} \quad (N_{Rd} = Af_o/\gamma_{M1}) \qquad B_0 = \frac{M_{0,Ed}}{M_{Rd}}$$

$$K_c = \frac{N_{Ed}}{\chi N_{Rd}} \qquad B_{haz} = B_0 \frac{f_o/\gamma_{M1}}{\rho_{haz} f_a/\gamma_{M2}}$$

$$K_x = K_0 + (K_c - K_0) \sin\left(\frac{\pi x}{l_c}\right)$$

$$K_{haz} = \left(K_0 + (K_c - K_0) \sin\left(\frac{\pi x_{haz}}{l_c}\right)\right) \frac{f_o/\gamma_{M1}}{\rho_{haz} f_a/\gamma_{M2}}$$

Check $\quad K_c^{\xi} + B_0 < 1{,}00$

and $\quad K_{haz}^{\xi} + B_{haz} < 1{,}00$

TALAT alu — Training in Aluminium Application Technologies | Beam-column with cross weld | 2301.09.20

9.09 Columns with unfilled bolt-holes or cut-outs

The expressions 9.28 - 9.30 are used, except that ρ_{haz} in 9.31 and 9.34 is replaced by A_{net}/A_{gr} where A_{net} is the net section area, with deduction of holes and A_{gr} is the gross section area.

9.10 Varying applied bending moment

For a beam-column subjected to axial compression and bending moment where the maximum applied moment do not coincide with the section with the largest buckling deflection, then some equivalent bending moment is often used. In Eurocode 9 a method is recommended implying that, in principle, every section along the beam-column must be checked for the combination of axial force, first order bending moment and second order bending moment. The influence of the axial force, inclusive the second order bending moment, is included in the first term in the interaction formulae. In a section apart from the section with the largest buckling deflection, the second order bending moment is less. The first quotient in the interaction formulae is then reduced by dividing it with the factor

$$\omega_x = \frac{1}{\chi + (1-\chi)\sin\dfrac{\pi x_s}{l_c}} \tag{9.35}$$

where:

χ = χ_y or χ_z depending on buckling direction

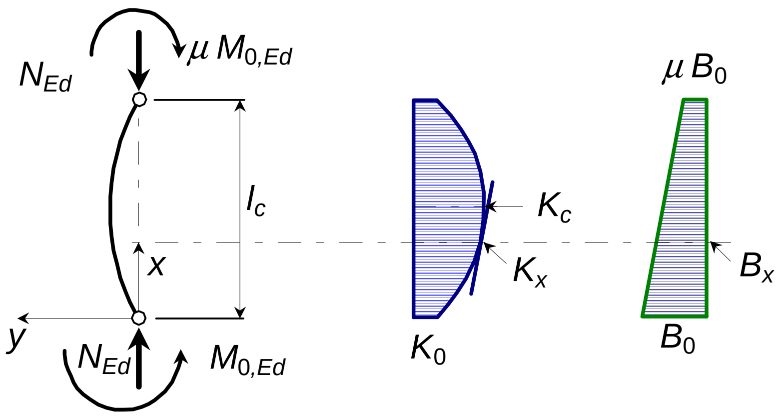
x_s = distance from the studied section to a pinned support or point of contra flexure for the deflection curve for elastic buckling, see **Figure 2301.09.19**

l_c = buckling length.

The method is illustrated in **Figure 2301.09.21** for a beam-column subjected to an axial compressive force and unequal end moments. Note that

$$\frac{N_{Ed}}{\omega_x \chi N_{Rd}} = \frac{N_{Ed}}{N_{Rd}} \; \frac{\chi + (1-\chi)\sin\dfrac{\pi x_s}{l_c}}{\chi} =$$

$$= \frac{N_{Ed}}{N_{Rd}} + \left(\frac{N_{Ed}}{\chi N_{Rd}} - \frac{N_{Ed}}{N_{Rd}}\right)\sin\frac{\pi x_s}{l_c} = K_0 + (K_c - K_0)\sin\frac{\pi x_s}{l_c} \tag{9.36}$$



$$K_x = K_0 + (K_c - K_0)\sin\left(\frac{\pi x}{l_c}\right)$$

$$B_x = B_0 - (B_0 - \mu\, B_0)\,\frac{x}{l_c}$$

$$K_0 = \frac{N_{Ed}}{N_{Rd}} \qquad K_c = \frac{N_{Ed}}{\chi\, N_{Rd}} \qquad B_0 = \frac{M_{0,Ed}}{M_{Rd}}$$

Check $K_x^{\xi} + B_x < 1{,}00$ in every section

TALAT Training in Aluminium Application Technologies	Beam-column with different end moments	2301.09.21

10 Deviation of linear stress distribution

10.01 General

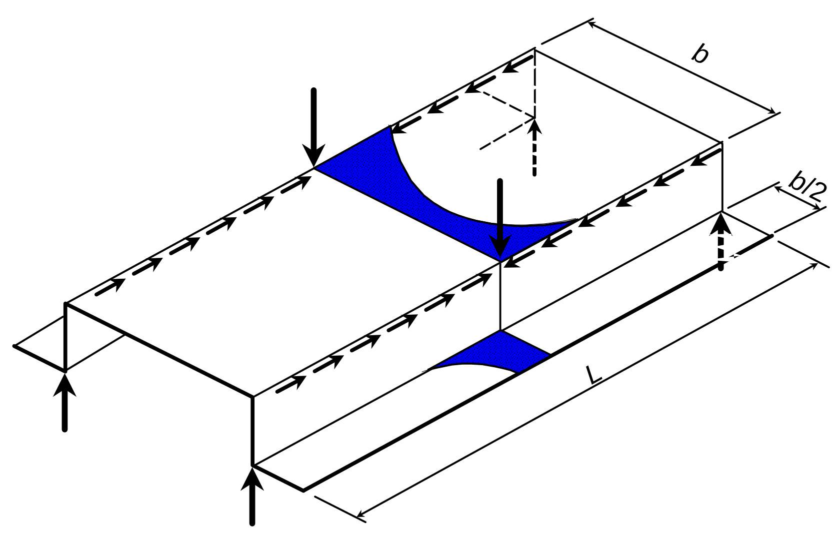
Deviation from linear stress distribution over the cross section may be caused by

- plastic deformation of cross sectional parts with a small slenderness ratio (small *b/t*)
- buckling of slender cross section parts (large *b/t*)
- shear deformations in a wide and short flange (small *L/b*), see 10.02
- flange curling of a wide slender flange (large *b/t*), see 10.03
- lateral deflection of non-symmetrical flanges, see 10.04
- introduction of concentrated loads
- variation in cross section and local rotations

Buckling and plastic deformations are dealt with in chapter 3. Local stress concentrations at load introduction, variation in cross section and rotations are normally determined by simplified calculation methods.

10.02 Shear lag

Shear stresses in the web are transferred to the flanges where they build up axial stresses. The stresses will not "reach" the centre of the flanges if a flange is wide and short because of the shear deformations. Consequently the stress distribution is as shown in **Figure 2301.10.01**.



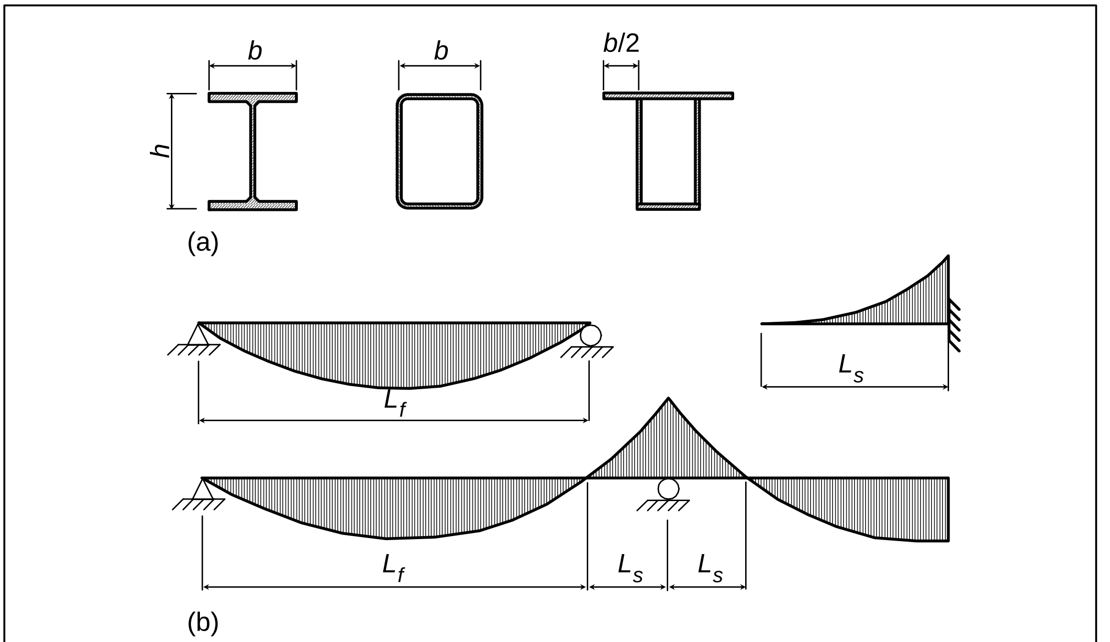
TALAT Training in Aluminium Application Technologies	Shear lag	2301.10.01

The influence of shear deflections depends on the relationship of flange width to beam length for the beam and shear force distribution along the beam. This effect exists for both the compression and the tension flange.

The influence of shear deflections for wide flanges may be allowed for by the effective width concept. If following conditions are fulfilled, the effective width is equal to the actual width.

$$b < L_f/10 \quad \text{and} \quad b < L_s/10$$

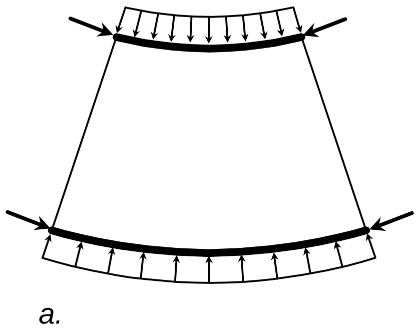
where b is the width of the flange according to **Figure 2301.10.02a** and L_s and L_f the distances according to **Figure 2301.10.02b**. See literature for effective width of flanges with large *b/L*.



TALAT Training in Aluminium Application Technologies | Notations | 2301.10.02

10.03 Flange curling of a wide flange

The bending stresses in the flanges of a beam contain a component directed towards the neutral axis, due to the curvature, cf. **Figure 2301.10.03a** (valid if the beam is straight before loading). This component is small for beams that are straight before loading and can be neglected for common rolled and extruded beams.



TALAT Training in Aluminium Application Technologies	Flange curling	2301.10.03

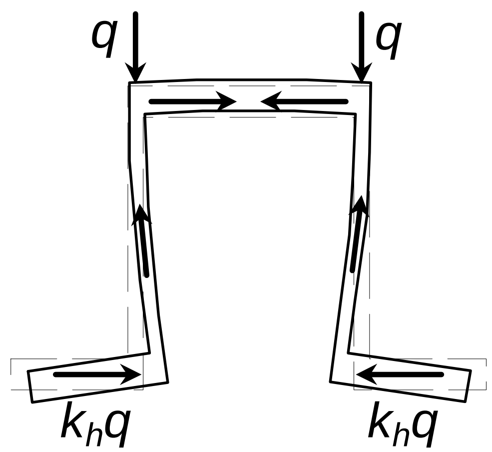
For wide, thin flanges and for curved beams the component causes bending of the flanges in the transverse direction. See literature.

For open cross section with small relation between depth and width the component above may cause transverse compression of the cross section.

10.04 Lateral deflection of non-symmetrical flanges

For a non-symmetrical flange the shear stress gradient in the flange cause lateral forces on the flanges, cf. **Figure 2301.10.04**. These forces are considered when determining the location of the shear centre for normal extruded, rolled and welded beams without cross section deformations.

For beams with a thin web the lateral forces must be considered. Usually k_h is in the order of 0,3, which can be used to check bending of the webs. See examples.



TALAT Training in Aluminium Application Technologies	Forces on non-symmetrical flanges	2301.10.04

11 Examples

11.01 Software

The examples are worked out using the MathCad software in which some symbols have special meanings according to the following:

$x := 50.6 \cdot mm$	Assign value
$y \equiv 2.5 \cdot mm$	Global assignment
$x + y = 53.1 \circ mm$	Evaluate expressior
$a = b$	Boolean equals
0.5	Decimal point
$c := (1 \ \ 3 \ \ 2)$	Vector
$d := (2 \ \ 4 \ \ 3)$	Vector
$a := \overrightarrow{(c \cdot d)}$	Vectorize
$a = \begin{bmatrix} 2 & 12 & 6 \end{bmatrix}$	Result

11.02 Cross section constants

See separate file **TALAT 2301 – 11.02**

11.03 Serviceability limit state

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