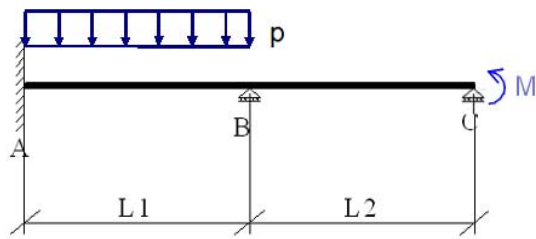


Løsningsforslag til opgave 10.3.2.



$$L_1 = \frac{2}{3} \cdot L_2$$

- A) Find reaktioner.
- B) Optegn snitkraftkurver for forskydning og moment.
- C) Kontrollér resultatet med et FEM software.

Stivhedsmatrice og "puttematrice" for element nr. 1

$$Ke := Es \cdot I_y \cdot \begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} ;; K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ;; K12 := \text{LinearAlgebra:-} \\
 \text{Transpose}(K11) ;; K1 := K11 \cdot Ke \cdot K12 :$$

Stivhedsmatrice og "puttematrice" for element nr. 2

$$\begin{aligned}
Ke := Es \cdot Iy \cdot & \begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} ; K21 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; K22 := \text{LinearAlgebra:-} \\
\text{Transpose}(K21) ; K2 := & K21 \cdot Ke \cdot K22 :
\end{aligned}$$

Samlet stivhedsmatrice og modificeret stivhedsmatrice det samlede system.

$$K := K1 + K2 ; p11 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} ; p12 := \text{LinearAlgebra:-Transpose}(p11) ; Kmod := p12 \cdot K \cdot p11 :$$

De virtuelle knudelaste fra den fordelte last.

$$\begin{aligned}
U1 := \int_0^{L1} \left(1 - \frac{3 \cdot x^2}{L1^2} + \frac{2 \cdot x^3}{L1^3} \right) \cdot p1 \, dx ; U2 := \int_0^{L1} - \left(x - \frac{2 \cdot x^2}{L1} + \frac{x^3}{L1^2} \right) \cdot p1 \, dx ; U33 := \int_0^{L1} \left(\frac{3 \cdot x^2}{L1^2} \right. \\
\left. - \frac{2 \cdot x^3}{L1^3} \right) \cdot p1 \, dx ; U4 := \int_0^{L1} \left(\frac{x^2}{L1} - \frac{x^3}{L1^2} \right) \cdot p1 \, dx :
\end{aligned}$$

$U1, U2, U33, U4$

$$\frac{1}{2} L1 p1, -\frac{1}{12} L1^2 p1, \frac{1}{2} L1 p1, \frac{1}{12} L1^2 p1 \tag{1}$$

Løsning af det modificerede system.

$$Umod := \begin{bmatrix} U4 \\ M \end{bmatrix} ; umod := Kmod^{-1} \cdot Umod :$$

Deformationer.

$$u := \begin{bmatrix} 0 \\ 0 \\ 0 \\ umod_1 \\ 0 \\ umod_2 \end{bmatrix} \quad ;: K.u :$$

$$Uf := \begin{bmatrix} U1 \\ U2 \\ U33 \\ U4 \\ 0 \\ 0 \end{bmatrix} :$$

De samlede knudelaste og reaktioner findes ved aftrække de virtuelle laster fra **k. u.**

$$U := K.u - Uf : \\ \xrightarrow{\text{simplify symbolic}}$$

$$\begin{bmatrix} -\frac{1}{2} \frac{5 L2 L1^2 p1 - 6 ML2 + 3 L1^3 p1}{L1 (4 L2 + 3 L1)} \\ \frac{1}{4} \frac{2 L2 L1^2 p1 - 4 ML2 + L1^3 p1}{4 L2 + 3 L1} \\ -\frac{1}{4} \frac{6 L2^2 L1^2 p1 + 12 L2^2 M + L1^4 p1 + 12 L1^2 M + 24 L2 L1 M + 6 L2 L1^3 p1}{L1 L2 (4 L2 + 3 L1)} \\ 0 \\ \frac{1}{4} \frac{L1^3 p1 + 12 ML1 + 24 ML2}{L2 (4 L2 + 3 L1)} \\ M \end{bmatrix} \quad (2)$$

$$\text{subs}(L2 = 1.5 \cdot L1, (2))$$

$$\begin{bmatrix} -\frac{0.055555555555 (10.5 LI^3 p1 - 9.0 MLI)}{LI^2} \\ \frac{0.027777777778 (4.0 LI^3 p1 - 6.0 MLI)}{LI} \\ -\frac{0.01851851852 (23.50 LI^4 p1 + 75.00 LI^2 M)}{LI^3} \\ 0 \\ \frac{0.01851851852 (LI^3 p1 + 48.0 MLI)}{LI^2} \\ M \end{bmatrix} \quad (3)$$

Taleksempel.

HEA200 profil, L1 = 4m og L2 = 6m. p1 = 20·kN/m.

$$I_y := 36900000 \cdot mm^4; E_s := \frac{210000 \cdot N}{mm^2}; L1 := 4 \cdot m; L2 := 6 \cdot m; p1 := \frac{20 \cdot kN}{m}; M := 30 \cdot kN \cdot m :$$

$$\begin{aligned} & 36900000 \text{ mm}^4 \\ & \frac{210000 \text{ N}}{mm^2} \\ & 4 \text{ m} \\ & 6 \text{ m} \\ & \frac{20 \text{ kN}}{m} \end{aligned} \quad (4)$$

U

$$\begin{bmatrix} -\frac{515}{12} \text{ kN} \\ \frac{275}{9} \text{ kN m} \\ -\frac{4885}{108} \text{ kN} \\ 0 \\ \frac{220}{27} \text{ kN} \\ 30 \text{ kN m} \end{bmatrix} \quad (5)$$

Reaktioner fra FEM-Design.



Momentkurve fra FEM-Design.

