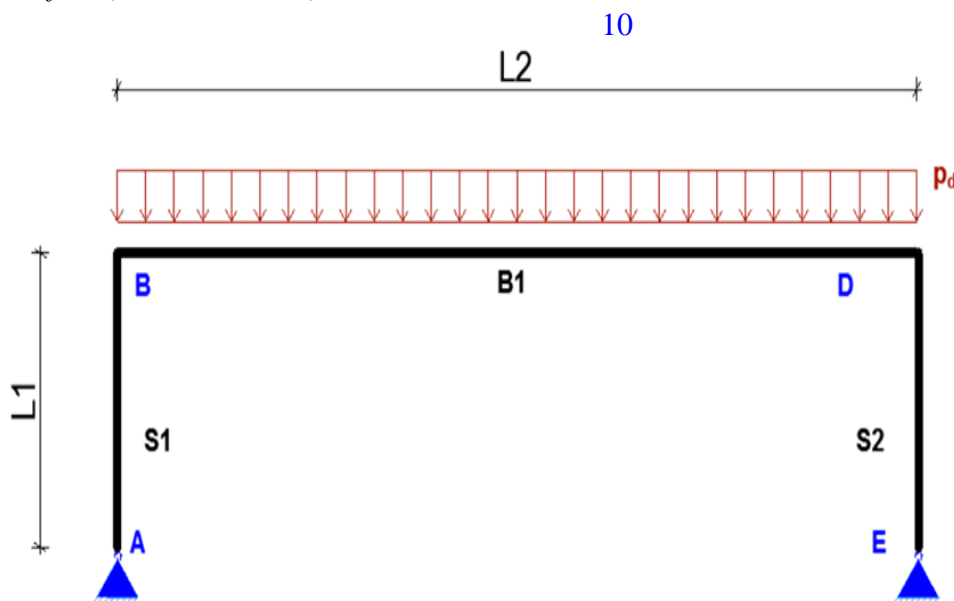


Opgave 4.51. Løsningsforslag. 2 charniers ramme med fordelt last.

```
[> restart
Loading LinearAlgebra
UseSystem(system, opts)
```

```
with(LinearAlgebra) :
interface(rtuplesize = 12)
```



(1)

Forudsætninger

$$\begin{aligned} \varphi 1 &:= 2 \cdot \left(\frac{x}{L}\right)^3 - 3 \cdot \left(\frac{x}{L}\right)^2 + 1 : \\ \varphi 2 &:= -x \cdot \left(\left(\frac{x}{L}\right)^2 - 2 \cdot \left(\frac{x}{L}\right) + 1 \right) : \\ \varphi 3 &:= -2 \cdot \left(\frac{x}{L}\right)^3 + 3 \cdot \left(\frac{x}{L}\right)^2 : \\ \varphi 4 &:= -x \cdot \left(\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right) \right) : \\ L &:= L2 \end{aligned}$$

$L2$

(1.1)

Stivhedsmatrice for et ramme element:

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12IE}{L^3} & -\frac{6IE}{L^2} & 0 & -\frac{12IE}{L^3} & -\frac{6IE}{L^2} \\ 0 & -\frac{6IE}{L^2} & \frac{4IE}{L} & 0 & \frac{6IE}{L^2} & \frac{2IE}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12IE}{L^3} & \frac{6IE}{L^2} & 0 & \frac{12IE}{L^3} & \frac{6IE}{L^2} \\ 0 & -\frac{6IE}{L^2} & \frac{2IE}{L} & 0 & \frac{6IE}{L^2} & \frac{4IE}{L} \end{bmatrix} :$$

$$T = \begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\nu) & \sin(\nu) & 0 \\ 0 & 0 & 0 & -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

▼ Stivhedsmatricer lokalt

$$K_{II} := \begin{bmatrix} \frac{EAI}{LI} & 0 & 0 & -\frac{EAI}{LI} & 0 & 0 \\ 0 & \frac{12EII}{LI^3} & -\frac{6EII}{LI^2} & 0 & -\frac{12EII}{LI^3} & -\frac{6EII}{LI^2} \\ 0 & -\frac{6EII}{LI^2} & \frac{4EII}{LI} & 0 & \frac{6EII}{LI^2} & \frac{2EII}{LI} \\ -\frac{EAI}{LI} & 0 & 0 & \frac{EAI}{LI} & 0 & 0 \\ 0 & -\frac{12EII}{LI^3} & \frac{6EII}{LI^2} & 0 & \frac{12EII}{LI^3} & \frac{6EII}{LI^2} \\ 0 & -\frac{6EII}{LI^2} & \frac{2EII}{LI} & 0 & \frac{6EII}{LI^2} & \frac{4EII}{LI} \end{bmatrix} :$$

$$K_{21} := \begin{bmatrix} \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 \\ 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} \\ -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 \\ 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2} \end{bmatrix} :$$

$$K_{31} := \begin{bmatrix} \frac{EA1}{L1} & 0 & 0 & -\frac{EA1}{L1} & 0 & 0 \\ 0 & \frac{12EI1}{L1^3} & -\frac{6EI1}{L1^2} & 0 & -\frac{12EI1}{L1^3} & -\frac{6EI1}{L1^2} \\ 0 & -\frac{6EI1}{L1^2} & \frac{4EI1}{L1} & 0 & \frac{6EI1}{L1^2} & \frac{2EI1}{L1} \\ -\frac{EA1}{L1} & 0 & 0 & \frac{EA1}{L1} & 0 & 0 \\ 0 & -\frac{12EI1}{L1^3} & \frac{6EI1}{L1^2} & 0 & \frac{12EI1}{L1^3} & \frac{6EI1}{L1^2} \\ 0 & -\frac{6EI1}{L1^2} & \frac{2EI1}{L1} & 0 & \frac{6EI1}{L1^2} & \frac{4EI1}{L1} \end{bmatrix} :$$

▼ **Stivhedsmatricer globalt**

$$T := \begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\nu) & \sin(\nu) & 0 \\ 0 & 0 & 0 & -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$$v1 := \frac{1}{2} \pi$$

$$T1 := \begin{bmatrix} \cos(v1) & \sin(v1) & 0 & 0 & 0 & 0 \\ -\sin(v1) & \cos(v1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(v1) & \sin(v1) & 0 \\ 0 & 0 & 0 & -\sin(v1) & \cos(v1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} : \frac{1}{2} \pi \quad (3.1)$$

$$K12 := T1^+ . K11 . T1 :$$

$$v2 := 0$$

$$0 \quad (3.2)$$

$$v3 := -\frac{\pi}{2}$$

$$T3 := \begin{bmatrix} \cos(v3) & \sin(v3) & 0 & 0 & 0 & 0 \\ -\sin(v3) & \cos(v3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(v3) & \sin(v3) & 0 \\ 0 & 0 & 0 & -\sin(v3) & \cos(v3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} : -\frac{1}{2} \pi \quad (3.3)$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

$$K32 := T3^+ . K31 . T3$$

$$\begin{bmatrix} \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} \\ 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\ 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} \end{bmatrix}$$

(3.5)

▼ Det samlede system

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; P_2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; P_3 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$$K1 := P_1 \cdot K12 \cdot P_1^+$$

$$\begin{bmatrix}
 \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
 -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}
 \tag{4.1}$$

$$K2 := P_2 \cdot K21 \cdot P_2^+$$

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & 0 \\
 0 & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}$$

(4.2)

$$K3 := P_3 \cdot K32 \cdot P_3^+$$

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} \\
 0 & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & -\frac{EA}{L} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\
 0 & 0 & 0 & 0 & 0 & 0 & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L}
 \end{bmatrix}$$

(4.3)

[Den samlede stivhedsmatrice

[> $K := K1 + K2 + K3$:

$$u := \begin{bmatrix} 0 \\ 0 \\ r1 \\ u2v \\ u2l \\ r2 \\ u3v \\ u3l \\ r3 \\ 0 \\ 0 \\ r4 \end{bmatrix} :$$

$$Uvir := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \int_0^{L2} \varphi1 \cdot pd \, dx \\ \int_0^{L2} \varphi2 \cdot pd \, dx \\ 0 \\ \int_0^{L2} \varphi3 \cdot pd \, dx \\ \int_0^{L2} \varphi4 \cdot pd \, dx \\ 0 \\ 0 \\ 0 \end{bmatrix} :$$

▼ **Modificeret system**

$$pm := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$$Kmod := pm^+ \cdot K \cdot pm$$

$$\left[\left[\frac{4EI}{L}, -\frac{6EI}{L^2}, 0, \frac{2EI}{L}, 0, 0, 0, 0 \right] \right]$$

(5.1)

$$\begin{aligned}
& \left[-\frac{6EI}{L^2}, \frac{12EI}{L^3} + \frac{EA}{L}, 0, -\frac{6EI}{L^2}, -\frac{EA}{L}, 0, 0, 0 \right], \\
& \left[0, 0, \frac{EA}{L} + \frac{12EI}{L^3}, -\frac{6EI}{L^2}, 0, -\frac{12EI}{L^3}, -\frac{6EI}{L^2}, 0 \right], \\
& \left[\frac{2EI}{L}, -\frac{6EI}{L^2}, -\frac{6EI}{L^2}, \frac{4EI}{L} + \frac{4EI}{L}, 0, \frac{6EI}{L^2}, \frac{2EI}{L}, 0 \right], \\
& \left[0, -\frac{EA}{L}, 0, 0, \frac{12EI}{L^3} + \frac{EA}{L}, 0, -\frac{6EI}{L^2}, -\frac{6EI}{L^2} \right], \\
& \left[0, 0, -\frac{12EI}{L^3}, \frac{6EI}{L^2}, 0, \frac{EA}{L} + \frac{12EI}{L^3}, \frac{6EI}{L^2}, 0 \right], \\
& \left[0, 0, -\frac{6EI}{L^2}, \frac{2EI}{L}, -\frac{6EI}{L^2}, \frac{6EI}{L^2}, \frac{4EI}{L} + \frac{4EI}{L}, \frac{2EI}{L} \right], \\
& \left[0, 0, 0, 0, -\frac{6EI}{L^2}, 0, \frac{2EI}{L}, \frac{4EI}{L} \right]
\end{aligned}$$

$$U_{mod} := \begin{bmatrix} U_{vir_3} \\ U_{vir_4} \\ U_{vir_5} \\ U_{vir_6} \\ U_{vir_7} \\ U_{vir_8} \\ U_{vir_9} \\ U_{vir_{12}} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} L2 pd \\ -\frac{1}{12} L2^2 pd \\ 0 \\ \frac{1}{2} L2 pd \\ \frac{1}{12} L2^2 pd \\ 0 \end{bmatrix}$$

(5.2)

$$umod := Kmod^{-1} \cdot Umod :$$

$$u1 := \begin{bmatrix} 0 \\ 0 \\ umod_1 \\ umod_2 \\ umod_3 \\ umod_4 \\ umod_5 \\ umod_6 \\ umod_7 \\ 0 \\ 0 \\ umod_8 \end{bmatrix} :$$

$$\begin{aligned} &evalf_3(u1) : \\ &evalf_3(K \cdot u1 - Uvir) : \end{aligned}$$

▼ **Eksempel. Samme profil som bjælke og søjler.**

$$L2 := 2 \cdot L1$$

$$2 L1$$

(6.1)

$$\begin{aligned}
& + \frac{1}{144} \left. \left(\frac{(16 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \right) \right] \\
& \left[- \frac{1}{L I^2} \left(6 E I I \left(- \frac{1}{144} \frac{(40 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{(8 A I I I L I^3 + 6 I I^2 L I) I I A I E} \right. \right. \right. \\
& + \frac{1}{144} \frac{(88 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{(8 A I I I L I^3 + 6 I I^2 L I) I I A I E} \left. \left. \left. \right) \right) + \left(\frac{12 E I I}{L I^3} \right. \right. \\
& + \frac{1}{2} \frac{E A I}{L I} \left. \left. \right) \left(- \frac{1}{144} \frac{(64 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) L I p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \right. \right. \\
& + \frac{1}{144} \frac{(64 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) L I p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \left. \left. \right) - \frac{1}{L I^2} \left(6 E I I \left(\right. \right. \right. \\
& - \frac{1}{144} \frac{(112 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \\
& + \frac{1}{144} \frac{(16 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \left. \left. \right) \right) - \frac{1}{2} \frac{1}{L I} \left(E A I \left(\right. \right. \\
& - \frac{1}{144} \frac{(64 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) L I p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \\
& + \frac{1}{144} \frac{(64 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) L I p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \left. \left. \right) \right) \left. \right] \\
& \left[\frac{\left(\frac{E A I}{L I} + \frac{3}{2} \frac{E I I}{L I^3} \right) L I^2 p d}{A I E} - \frac{3}{2} \frac{1}{L I^2} \left(E I I \left(\right. \right. \right. \\
& - \frac{1}{144} \frac{(112 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \\
& + \frac{1}{144} \frac{(16 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \left. \left. \right) \right) - \frac{3}{2} \frac{I I p d}{L I A I} \\
& - \frac{3}{2} \frac{1}{L I^2} \left(E I I \left(- \frac{1}{144} \frac{(16 A I^2 I I L I^6 + 96 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \right. \right. \\
& + \frac{1}{144} \frac{(112 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{E A I I I (8 A I I I L I^3 + 6 I I^2 L I)} \left. \left. \right) \right) - L I p d \left. \right] \\
\end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{LI} \left(2EI \left(-\frac{1}{144} \frac{(40AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) pd}{(8AI I LI^3 + 6I^2 LI) IAI E} \right. \right. \right. \\
& + \frac{1}{144} \frac{(88AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) pd}{(8AI I LI^3 + 6I^2 LI) IAI E} \left. \left. \right) \right) - \frac{1}{LI^2} \left(6EI \left(\right. \right. \\
& - \frac{1}{144} \frac{(64AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \\
& + \frac{1}{144} \frac{(64AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \left. \left. \right) \right) + \frac{1}{LI} \left(6EI \left(\right. \right. \\
& - \frac{1}{144} \frac{(112AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \\
& + \frac{1}{144} \frac{(16AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \left. \left. \right) \right) + \frac{1}{LI} \left(EI \left(\right. \right. \\
& - \frac{1}{144} \frac{(16AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \\
& + \frac{1}{144} \frac{(112AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \left. \left. \right) \right) + \frac{1}{3} LI^2 pd \Big], \\
& \left[-\frac{1}{2} \frac{1}{LI} \left(EAI \left(-\frac{1}{144} \frac{(64AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \right. \right. \right. \\
& + \frac{1}{144} \frac{(64AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \left. \left. \right) \right) + \left(\frac{12EI}{LI^3} \right. \\
& + \frac{1}{2} \frac{EAI}{LI} \left. \right) \left(-\frac{1}{144} \frac{(64AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \right. \\
& + \frac{1}{144} \frac{(64AI^2 I LI^6 + 384AI I^2 LI^4 + 144I^3 LI^2) LI pd}{EAI I (8AI I LI^3 + 6I^2 LI)} \left. \right) - \frac{1}{LI^2} \left(6EI \left(\right. \right. \\
& - \frac{1}{144} \frac{(16AI^2 I LI^6 + 96AI I^2 LI^4 + 144I^3 LI^2) pd}{EAI I (8AI I LI^3 + 6I^2 LI)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \Big) - \frac{1}{LI^2} \left(6 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(88 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II AI E} \\
& \left. \left. + \frac{1}{144} \frac{(40 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II AI E} \right) \right) \Big],
\end{aligned}$$

$$\begin{aligned}
& \left[- \frac{3}{2} \frac{II pd}{LI AI} + \frac{3}{2} \frac{1}{LI^2} \left(E II \left(\right. \right. \right. \\
& - \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \\
& \left. \left. + \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \right) \right) \\
& + \frac{\left(\frac{E AI}{LI} + \frac{3}{2} \frac{E II}{LI^3} \right) LI^2 pd}{AI E} + \frac{3}{2} \frac{1}{LI^2} \left(E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \\
& \left. \left. + \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \right) \right) - LI pd \Big],
\end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{LI} \left(E II \left(- \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \right. \right. \right. \\
& \left. \left. + \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \right) \right) - \frac{1}{LI^2} \left(6 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(64 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \\
& \left. \left. + \frac{1}{144} \frac{(64 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)} \right) \right) + \frac{1}{LI} \left(6 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{E AI II (8 AI II LI^3 + 6 II^2 LI)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \Big) + \frac{1}{LI} \left(2 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(88 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II A I E} \\
& + \frac{1}{144} \frac{(40 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II A I E} \Big) \Big) - \frac{1}{3} LI^2 pd \Big], \\
& \left[- \frac{1}{LI^3} \left(12 E II \left(- \frac{1}{144} \frac{(64 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \right. \right. \right. \\
& + \frac{1}{144} \frac{(64 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \Big) \Big) + \frac{1}{LI^2} \left(6 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \\
& + \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \Big) \Big) + \frac{1}{LI^2} \left(6 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(88 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II A I E} \\
& + \frac{1}{144} \frac{(40 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II A I E} \Big) \Big) \Big], \\
& \left[-LI pd \right], \\
& \left[- \frac{1}{LI^2} \left(6 E II \left(- \frac{1}{144} \frac{(64 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \right. \right. \right. \\
& + \frac{1}{144} \frac{(64 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) LI pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \Big) \Big) + \frac{1}{LI} \left(2 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(16 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \\
& + \frac{1}{144} \frac{(112 AI^2 II LI^6 + 384 AI II^2 LI^4 + 144 II^3 LI^2) pd}{EAI II (8 AI II LI^3 + 6 II^2 LI)} \Big) \Big) + \frac{1}{LI} \left(4 E II \left(\right. \right. \\
& - \frac{1}{144} \frac{(88 AI^2 II LI^6 + 96 AI II^2 LI^4 + 144 II^3 LI^2) pd}{(8 AI II LI^3 + 6 II^2 LI) II A I E}
\end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{144} \frac{(40 A I^2 I I L I^6 + 384 A I I I^2 L I^4 + 144 I I^3 L I^2) p d}{(8 A I I I L I^3 + 6 I I^2 L I) I I A I E} \Big) \Big) \Big] \\
 \text{simplify} \\
 = & \left[\begin{array}{c} -\frac{L I^3 p d A I}{4 A I L I^2 + 3 I I} \\ -L I p d \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{L I^3 p d A I}{4 A I L I^2 + 3 I I} \\ -L I p d \\ 0 \end{array} \right] \tag{6.5}
 \end{aligned}$$

Som det ses afhænger den vandrette reaktion både af inertimoment og areal.

Eksempel 1. HEA200:

$$A1 := 5380 \tag{7.1}$$

$$A2 := A1 : \tag{7.2}$$

$$I I := 36.9 \cdot 10^6 \tag{7.2}$$

$$I2 := I I : \tag{7.3}$$

$$E := 210000 \tag{7.3}$$

$$L1 := 5000 \tag{7.4}$$

$$L2 := 2 \cdot L1 \tag{7.4}$$

$$v1 \tag{7.5}$$

$$\frac{1}{2} \pi \tag{7.6}$$

$$v2 \tag{7.7}$$

$v3$

$$-\frac{1}{2} \pi \quad (7.8)$$

$pd := 10$

$$10 \quad (7.9)$$

$u1 :$

$K.u1 - Uvir$

$$\begin{bmatrix} -12497.42849 \\ -50000. \\ 0.01 \\ -0.003347 \\ 0. \\ -0.06 \\ 0.00335 \\ 0. \\ 0.01 \\ 12497.42849 \\ -50000. \\ -0.01 \end{bmatrix} \quad (7.10)$$

Teknisk Ståbi

$$H := \frac{pd \cdot L2^2}{4 \cdot LI \cdot \left(2 \cdot \frac{1}{2} + 3\right)}$$

$$12500 \quad (7.11)$$

$$L := \frac{1}{2} \cdot pd \cdot L2$$

$$50000 \quad (7.12)$$

Eksempel 2. HEA 1000:

$A1 := 34700$

$$34700 \quad (8.1)$$

$A2 := A1 :$

$I1 := 5538 \cdot 10^6$

$$5538000000 \quad (8.2)$$

$I2 := I1 :$

$E := 210000$

$$L1 := 5000 \quad 210000 \quad (8.3)$$

$$L2 := 2 \cdot L1 \quad 5000 \quad (8.4)$$

$$v1 \quad 10000 \quad (8.5)$$

$$v2 \quad \frac{1}{2} \pi \quad (8.6)$$

$$v3 \quad 0 \quad (8.7)$$

$$pd := 10 \quad -\frac{1}{2} \pi \quad (8.8)$$

$$u1 : \quad 10 \quad (8.9)$$

$K.u1 - Uvir$

$$\left[\begin{array}{c} -\frac{21687500000}{1743307} \\ -50000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{21687500000}{1743307} \\ -50000 \\ 0 \end{array} \right] \quad (8.10)$$

at 5 digits →

$$\begin{bmatrix} -12440. \\ -50000. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 12440. \\ -50000. \\ 0. \end{bmatrix}$$

(8.11)

Teknisk Ståbi

$$H := \frac{pd \cdot L^2}{4 \cdot L \cdot \left(2 \cdot \frac{1}{2} + 3\right)}$$

12500

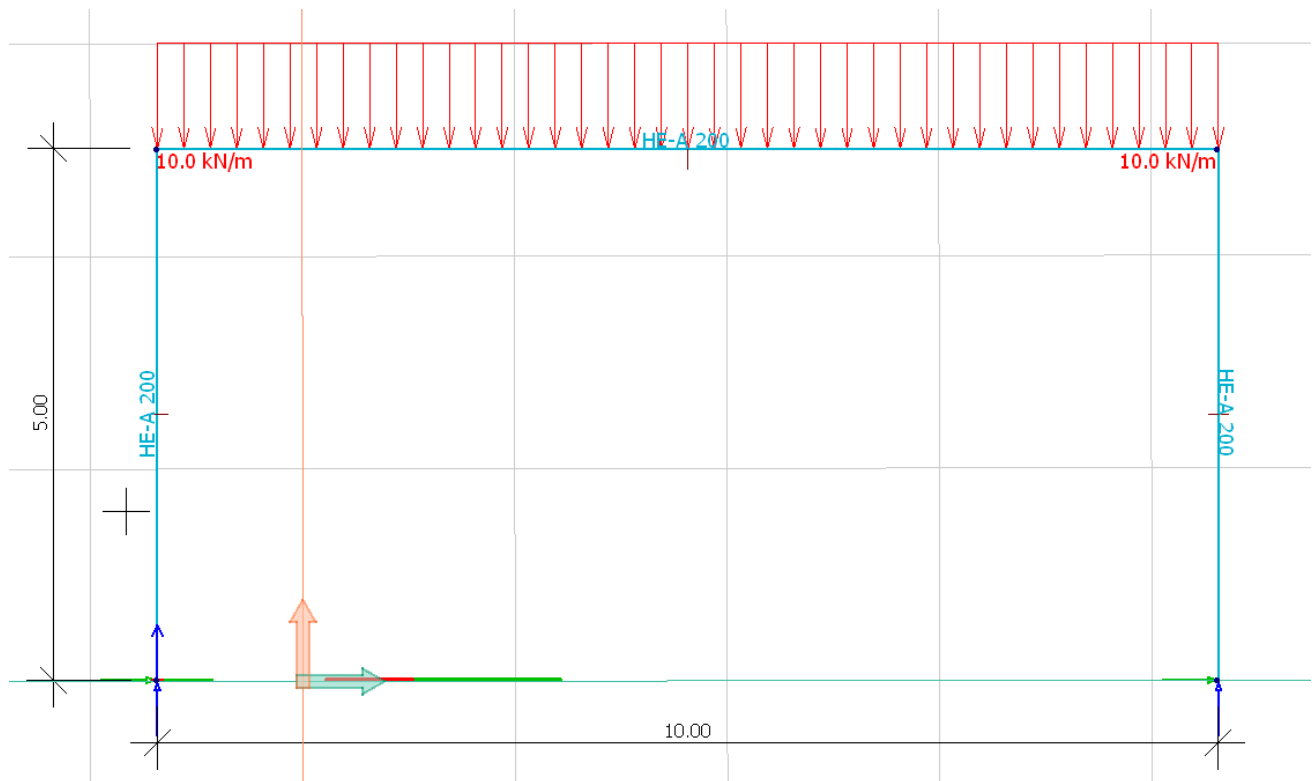
(8.12)

$$L := \frac{1}{2} \cdot pd \cdot L^2$$

50000

(8.13)

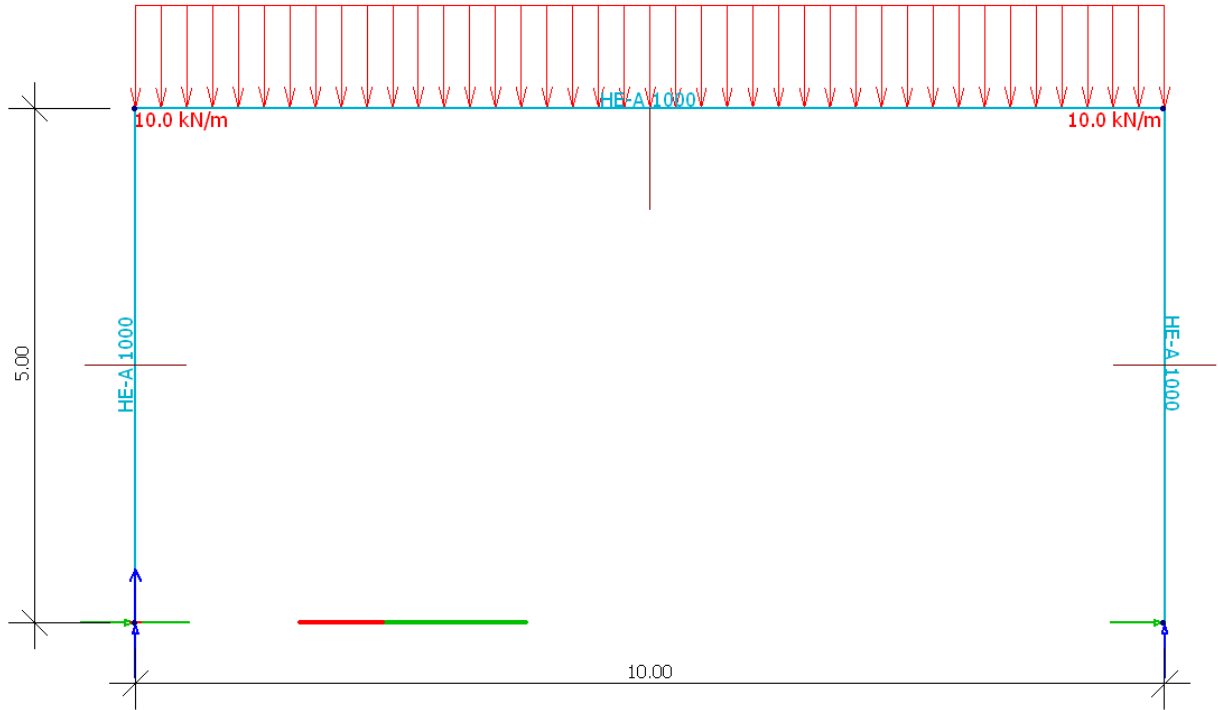
▼ **FEM.Design. Eksempel 1. HEA200**



Point support group, reactions, Load comb.: Lodret

ID	x	y	z	Node	Fx'	Fy'	Fz'	Mx'	My'	Mz'
[-]	[m]	[m]	[m]	[-]	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
S.1	0.000	0.000	0.000	1	-12.468	0.000	-50.000	0.000	0.000	0.000
S.2	10.000	0.000	0.000	5	12.468	0.000	-50.000	0.000	0.000	0.000

FEM-Design. Eksempel 2. HEA1000



Point support group, reactions, Load comb.: Lodret

ID	x	y	z	Node	Fx'	Fy'	Fz'	Mx'	My'	Mz'
[-]	[m]	[m]	[m]	[-]	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
S.1	0.000	0.000	0.000	1	-12.113	0.000	-50.000	0.000	0.000	0.000
S.2	10.000	0.000	0.000	5	12.113	0.000	-50.000	0.000	0.000	0.000