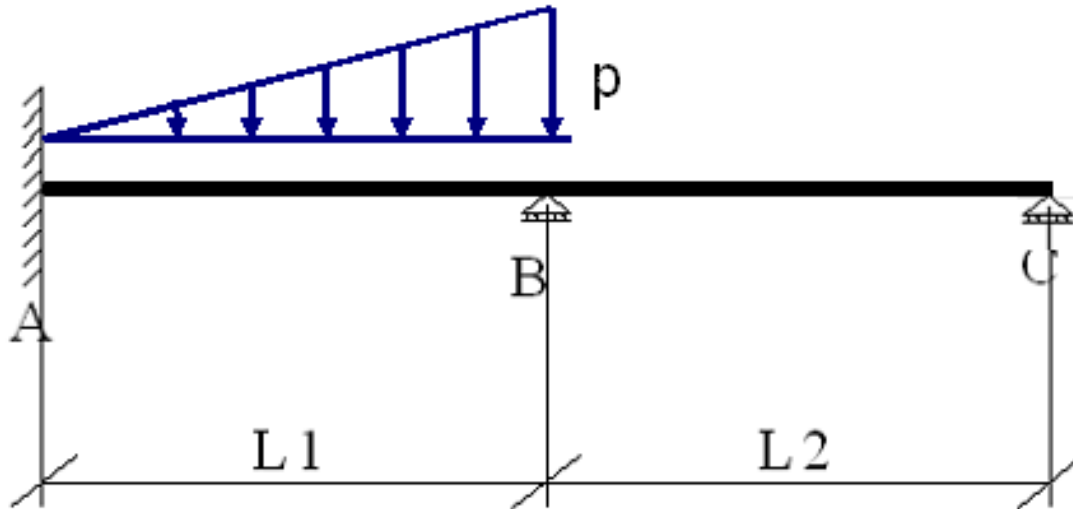


> with(Student[LinearAlgebra]) :

**Løsningsforslag til opgave 10.3.1**



>

>  $Ke := Es \cdot Iy \cdot$

$$\begin{bmatrix} \frac{12}{L_1^3} & -\frac{6}{L_1^2} & -\frac{12}{L_1^3} & -\frac{6}{L_1^2} \\ -\frac{6}{L_1^2} & \frac{4}{L_1} & \frac{6}{L_1^2} & \frac{2}{L_1} \\ -\frac{12}{L_1^3} & \frac{6}{L_1^2} & \frac{12}{L_1^3} & \frac{6}{L_1^2} \\ -\frac{6}{L_1^2} & \frac{2}{L_1} & \frac{6}{L_1^2} & \frac{4}{L_1} \end{bmatrix}$$

$$Ke := \begin{bmatrix} \frac{12 Es Iy}{L_1^3} & -\frac{6 Es Iy}{L_1^2} & -\frac{12 Es Iy}{L_1^3} & -\frac{6 Es Iy}{L_1^2} \\ -\frac{6 Es Iy}{L_1^2} & \frac{4 Es Iy}{L_1} & \frac{6 Es Iy}{L_1^2} & \frac{2 Es Iy}{L_1} \\ -\frac{12 Es Iy}{L_1^3} & \frac{6 Es Iy}{L_1^2} & \frac{12 Es Iy}{L_1^3} & \frac{6 Es Iy}{L_1^2} \\ -\frac{6 Es Iy}{L_1^2} & \frac{2 Es Iy}{L_1} & \frac{6 Es Iy}{L_1^2} & \frac{4 Es Iy}{L_1} \end{bmatrix}$$

(1)

$$\begin{aligned}
 > U1 := \int_0^{L1} \frac{\left(1 - \frac{3 \cdot x^2}{L1^2} + \frac{2 \cdot x^3}{L1^3}\right) \cdot p1 \cdot x}{L1} dx \\
 & U1 := \frac{3}{20} L1 p1 \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 > U2 := \int_0^{L1} \frac{-\left(x - \frac{2 \cdot x^2}{L1} + \frac{x^3}{L1^2}\right) \cdot p1 \cdot x}{L1} dx \\
 & U2 := -\frac{1}{30} L1^2 p1 \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 > U3 := \int_0^{L1} \frac{\left(\frac{3 \cdot x^2}{L1^2} - \frac{2 \cdot x^3}{L1^3}\right) \cdot p1 \cdot x}{L1} dx \\
 & U3 := \frac{7}{20} L1 p1 \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 > U4 := \int_0^{L1} \frac{\left(\frac{x^2}{L1} - \frac{x^3}{L1^2}\right) \cdot p1 \cdot x}{L1} dx \\
 & U4 := \frac{1}{20} L1^2 p1 \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 > K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
 & K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)
 \end{aligned}$$

> LinearAlgebra:-Transpose( (6) )

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (7)$$

>  $K12 := \text{LinearAlgebra:-Transpose}(K11)$

$$K12 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (8)$$

>  $K1 := K11 \cdot Ke \cdot K12$

$$K1 := \begin{bmatrix} \frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & 0 & 0 \\ -\frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & 0 & 0 \\ -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & 0 & 0 \\ -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

>

$$K21 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K21 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

>  $K22 := \text{LinearAlgebra:-Transpose}(K21)$

$$K22 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

>  $K2 := K21 \cdot Ke \cdot K22$

$$K2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} \\ 0 & 0 & -\frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} \\ 0 & 0 & -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} \\ 0 & 0 & -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} \end{bmatrix} \quad (12)$$

>  $K := K1 + K2$

$$K := \begin{bmatrix} \frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & 0 & 0 \\ -\frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & 0 & 0 \\ -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{24 Es Iy}{LI^3} & 0 & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} \\ -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & 0 & \frac{8 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} \\ 0 & 0 & -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} \\ 0 & 0 & -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} \end{bmatrix} \quad (13)$$

>  $p11 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$p11 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (14)$$

>  $p12 := \text{LinearAlgebra:-Transpose}(p11)$

$$p12 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

>  $Kmod := p12.K.p11$

$$Kmod := \begin{bmatrix} \frac{8 Es Iy}{Ll} & \frac{2 Es Iy}{Ll} \\ \frac{2 Es Iy}{Ll} & \frac{4 Es Iy}{Ll} \end{bmatrix} \quad (16)$$

>  $Umod := \begin{bmatrix} U4 \\ 0 \end{bmatrix}$

$$Umod := \begin{bmatrix} \frac{1}{20} Ll^2 p1 \\ 0 \end{bmatrix} \quad (17)$$

>

>  $Kmod^{-1}$

$$\begin{bmatrix} \frac{1}{7} \frac{Ll}{Es Iy} & -\frac{1}{14} \frac{Ll}{Es Iy} \\ -\frac{1}{14} \frac{Ll}{Es Iy} & \frac{2}{7} \frac{Ll}{Es Iy} \end{bmatrix} \quad (18)$$

>  $umod := Kmod^{-1}.Umod$

$$umod := \begin{bmatrix} \frac{1}{140} \frac{Ll^3 p1}{Es Iy} \\ -\frac{1}{280} \frac{Ll^3 p1}{Es Iy} \end{bmatrix} \quad (19)$$



$$\begin{bmatrix} -\frac{3}{70} L l p l \\ \frac{1}{70} L l^2 p l \\ \frac{3}{140} L l p l \\ \frac{1}{20} L l^2 p l \\ \frac{3}{140} L l p l \\ 0 \end{bmatrix} \quad (22)$$

$$\rightarrow U := K \cdot u - U_f$$

$$U := \begin{bmatrix} -\frac{27}{140} L l p l \\ \frac{1}{21} L l^2 p l \\ -\frac{23}{70} L l p l \\ 0 \\ \frac{3}{140} L l p l \\ 0 \end{bmatrix} \quad (23)$$

$$\rightarrow I_y := 36.92 \cdot 10^6 \cdot \text{mm}^4; E_s := \frac{210000 \cdot N}{\text{mm}^2}; L l := 4 \cdot m; p l := \frac{10 \cdot kN}{m}$$

$$I_y := 3.692000000 \cdot 10^7 \text{ mm}^4$$

$$E_s := \frac{210000 N}{\text{mm}^2}$$

$$L l := 4 m$$

$$p l := \frac{10 kN}{m} \quad (24)$$

$$\rightarrow U$$

(25)

$$\begin{bmatrix} -7,71 \text{ kN} \\ 7,62 \text{ m kN} \\ -13,14 \text{ kN} \\ 0,00 \\ 857,00 \times 10^{-3} \text{ kN} \\ 0,00 \end{bmatrix}$$

(25)

**Kontrol i FEM-Design. Som det ses, er der lidt regneøjagtighed i FEM-Design ( $19.696 = 1/2 \cdot 10 \text{ kN/m} \cdot 4$ )**

