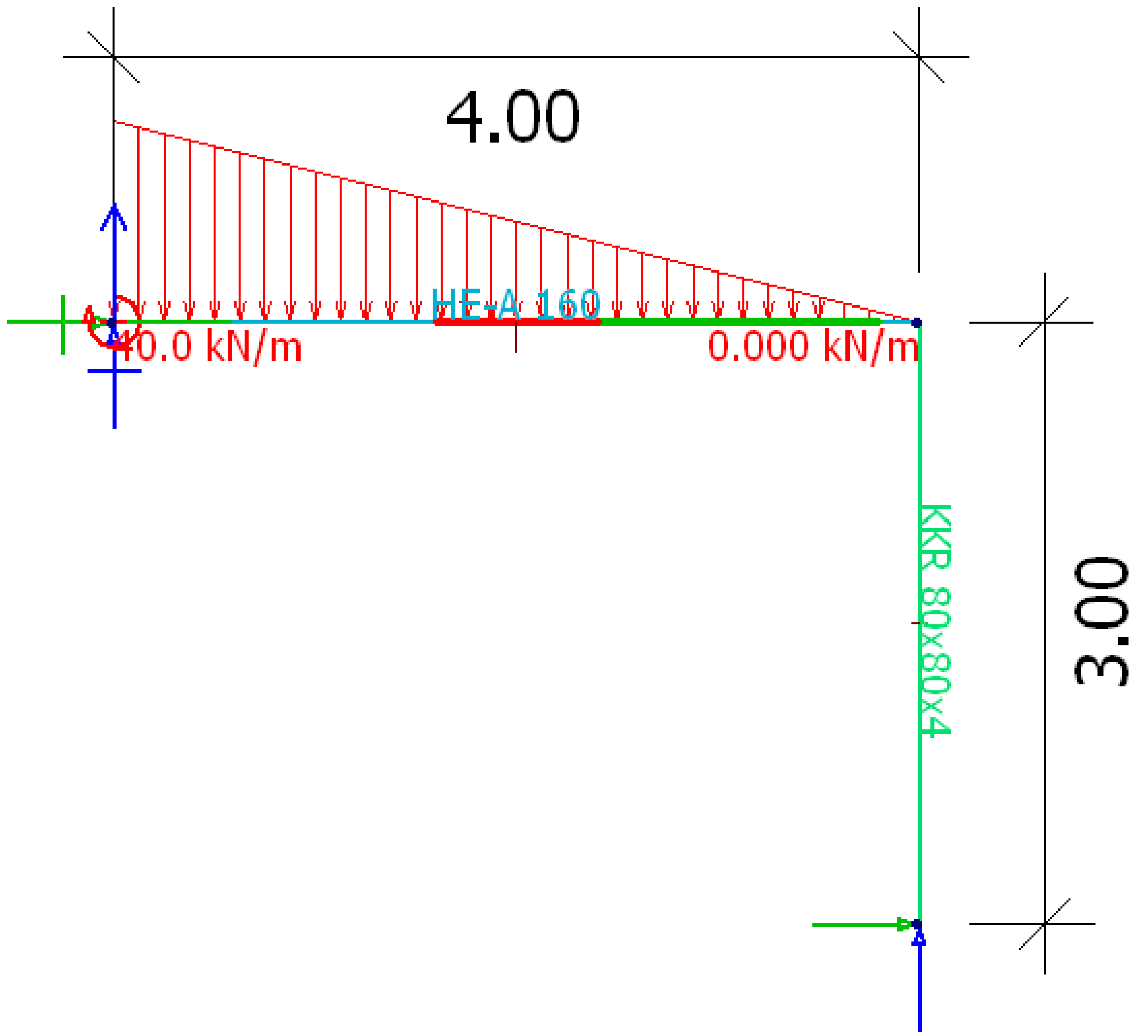
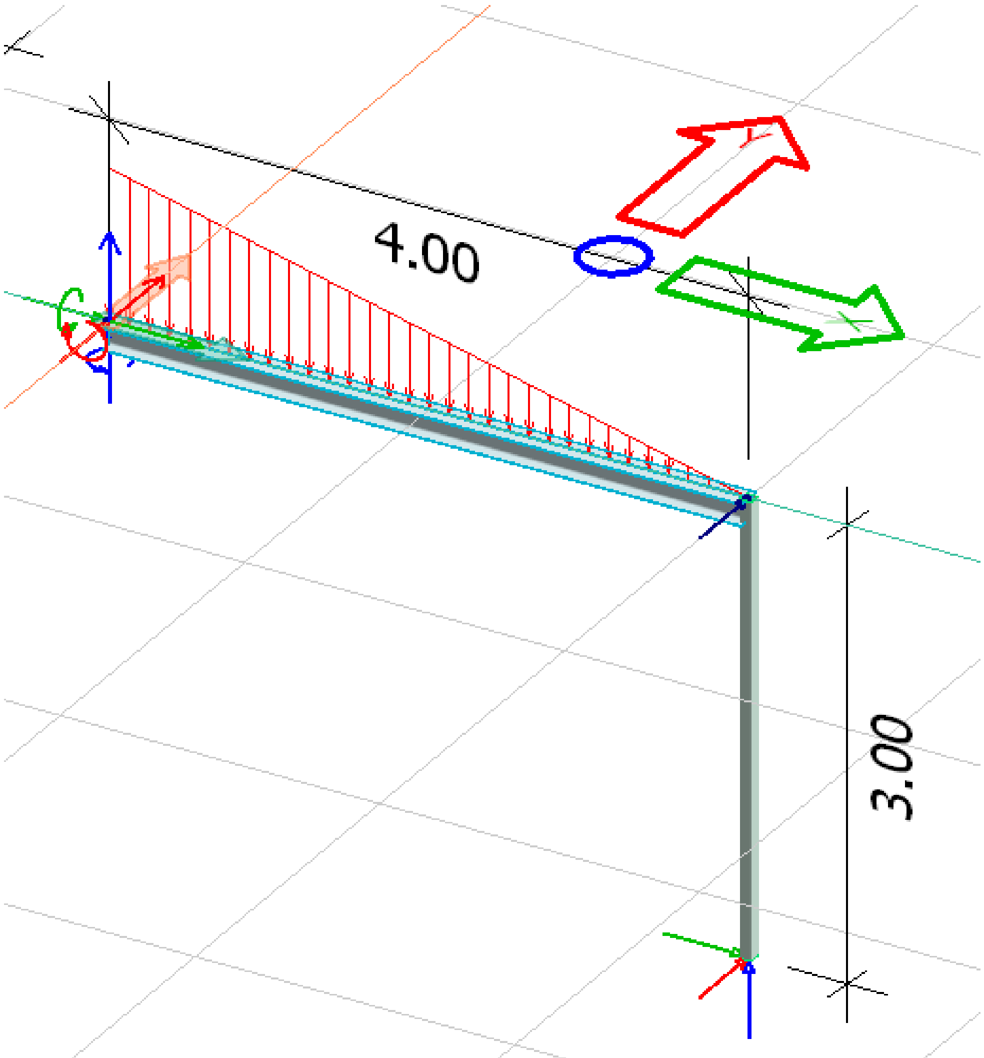
Løsningsforslag til opgave **11.2.1.**

with(*Student*[*LinearAlgebra*]) :
restart

Bjælke er HEA160 og fsøjlen er en KKR 80x80x4. Der regnes med charnier i samlingen mellem elementerne, hvilket her klares ved at nedsætte inertimomentet fir KKR til meget lille. Dette kan gøres, da der kun er normalkræfter i elementet. Efterfølgende ændres søjlen til HEA160 med momentstiv samling mellem bjælke og søjle.





Stivhedsmatrice for element 1

$$K11 := \begin{bmatrix} \frac{E\,A1}{L1} & 0 & 0 & -\frac{E\,A1}{L1} & 0 & 0 \\ 0 & \frac{12\,E\,I1}{L1^3} & -\frac{6\,E\,I1}{L1^2} & 0 & -\frac{12\,E\,I1}{L1^3} & -\frac{6\,E\,I1}{L1^2} \\ 0 & -\frac{6\,E\,I1}{L1^2} & \frac{4\,E\,I1}{L1} & 0 & \frac{6\,E\,I1}{L1^2} & \frac{2\,E\,I1}{L1} \\ -\frac{E\,A1}{L1} & 0 & 0 & \frac{E\,A1}{L1} & 0 & 0 \\ 0 & -\frac{12\,E\,I1}{L1^3} & \frac{6\,E\,I1}{L1^2} & 0 & \frac{12\,E\,I1}{L1^3} & \frac{6\,E\,I1}{L1^2} \\ 0 & -\frac{6\,E\,I1}{L1^2} & \frac{2\,E\,I1}{L1} & 0 & \frac{6\,E\,I1}{L1^2} & \frac{4\,E\,I1}{L1} \end{bmatrix} :$$

Puttematrice for K11

$$p1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}:$$

Stivhedsmtarice for element 2 i det lokale system

$$K12 := \begin{bmatrix} \frac{E\,A2}{L2} & 0 & 0 & -\frac{E\,A2}{L2} & 0 & 0 \\ 0 & \frac{12\,E\,I2}{L2^3} & -\frac{6\,E\,I2}{L2^2} & 0 & -\frac{12\,E\,I2}{L2^3} & -\frac{6\,E\,I2}{L2^2} \\ 0 & -\frac{6\,E\,I2}{L2^2} & \frac{4\,E\,I2}{L2} & 0 & \frac{6\,E\,I2}{L2^2} & \frac{2\,E\,I2}{L2} \\ -\frac{E\,A2}{L2} & 0 & 0 & \frac{E\,A2}{L2} & 0 & 0 \\ 0 & -\frac{12\,E\,I2}{L2^3} & \frac{6\,E\,I2}{L2^2} & 0 & \frac{12\,E\,I2}{L2^3} & \frac{6\,E\,I2}{L2^2} \\ 0 & -\frac{6\,E\,I2}{L2^2} & \frac{2\,E\,I2}{L2} & 0 & \frac{6\,E\,I2}{L2^2} & \frac{4\,E\,I2}{L2} \end{bmatrix}:$$

$$v := \frac{\pi}{2}$$

$$\frac{1}{2}\,\pi \qquad (1)$$

Transformationsmatrice for element 1

$$T := \begin{bmatrix} \cos(v) & \sin(v) & 0 & 0 & 0 & 0 \\ -\sin(v) & \cos(v) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(v) & \sin(v) & 0 \\ 0 & 0 & 0 & -\sin(v) & \cos(v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

$K22 := T^{+}.K12.T :$

Puttematrice for K2

$$p2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$K1 := p1.K11.p1^{+} :$

$K2 := p2.K22.p2^{+} :$

Samlet stivhedsmatrice

$K := K1 + K2 :$

$$u := \begin{bmatrix} 0 \\ 0 \\ 0 \\ ubv \\ ubl \\ rb \\ 0 \\ 0 \\ r_C \end{bmatrix} :$$

$$Ur := \begin{bmatrix} Av \\ Al \\ MA \\ 0 \\ 0 \\ 0 \\ Cv \\ Cl \\ 0 \end{bmatrix} :$$

De virtuelle knudelaste fra den fordelte last.

$p1 := \left(1 - \frac{x}{L1}\right) \cdot q :$

$$U1 := \int_0^{L1} \left(1 - \frac{3 \cdot x^2}{L1^2} + \frac{2 \cdot x^3}{L1^3}\right) \cdot p1 \, \mathrm{d}x :; U2 := \int_0^{L1} -\left(x - \frac{2 \cdot x^2}{L1} + \frac{x^3}{L1^2}\right) \cdot p1 \, \mathrm{d}x :; U3 := \int_0^{L1} \left(\frac{3 \cdot x^2}{L1^2} - \frac{2 \cdot x^3}{L1^3}\right) \cdot p1 \, \mathrm{d}x :;$$

$$U4 := \int_0^{L1} \left(\frac{x^2}{L1} - \frac{x^3}{L1^2} \right) \cdot p1 \, dx :$$

$U1, U2, U3, U4$

$$\frac{7}{20}\, L1\, q,\ -\frac{1}{20}\, L1^2\, q,\ \frac{3}{20}\, L1\, q,\ \frac{1}{30}\, L1^2\, q \tag{3}$$

$$Uv := \begin{bmatrix} 0 \\ U1 \\ U2 \\ 0 \\ U3 \\ U4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \frac{7}{20}\, L1\, q \\ -\frac{1}{20}\, L1^2\, q \\ 0 \\ \frac{3}{20}\, L1\, q \\ \frac{1}{30}\, L1^2\, q \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$U := Ur + Uv$

$$\begin{bmatrix} Av \\ Al + \frac{7}{20}\, L1\, q \\ MA - \frac{1}{20}\, L1^2\, q \\ 0 \\ \frac{3}{20}\, L1\, q \\ \frac{1}{30}\, L1^2\, q \\ Cv \\ Cl \\ 0 \end{bmatrix} \tag{5}$$

Løsning af det modificerede system.

Puttematrice pmod

$$pmod := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} :$$

$Kmod := pmod^{+}.K.pmod$

$$\begin{bmatrix} \frac{E\,A1}{L1} + \frac{12\,E\,I2}{L2^3} & 0 & \frac{6\,E\,I2}{L2^2} & \frac{6\,E\,I2}{L2^2} \\ 0 & \frac{12\,E\,I1}{L1^3} + \frac{E\,A2}{L2} & \frac{6\,E\,I1}{L1^2} & 0 \\ \frac{6\,E\,I2}{L2^2} & \frac{6\,E\,I1}{L1^2} & \frac{4\,E\,I1}{L1} + \frac{4\,E\,I2}{L2} & \frac{2\,E\,I2}{L2} \\ \frac{6\,E\,I2}{L2^2} & 0 & \frac{2\,E\,I2}{L2} & \frac{4\,E\,I2}{L2} \end{bmatrix} \quad \textbf{(6)}$$

$$Umod := \begin{bmatrix} U_4 \\ U_5 \\ U_6 \\ U_9 \end{bmatrix} ; umod := Kmod^{-1}.Umod :$$

$$\begin{bmatrix} 0 \\ \frac{3}{20}\,L1\,q \\ \frac{1}{30}\,L1^2\,q \\ 0 \end{bmatrix} \quad \textbf{(7)}$$

$$u := \begin{bmatrix} 0 \\ 0 \\ 0 \\ umod_1 \\ umod_2 \\ umod_3 \\ 0 \\ 0 \\ umod_4 \end{bmatrix} :$$

$U := K.u$:

$UR := U - Uv$:

Taleksempel 1.
Element 1 er HEA160 og element 2 er KKR 80x80x4.

$I1 := 16700000; E := 210000; L1 := 4000; A1 := 3877; L2 := 3000; q := 40; I2 := \frac{I1}{10000000}; A2 := 1175$

$$16700000$$
$$210000$$
$$4000$$
$$3877$$
$$3000$$
$$40$$
$$\frac{167}{100}$$
$$1175 \quad (8)$$

$\frac{E \cdot A2}{L2}$

$$82250 \quad (9)$$

$UR := U - Uv$:
$evalf_3(UR \cdot 1.0)$

$$\begin{bmatrix} 0.000703 \\ -64000. \\ 4.28\,10^7 \\ 0. \\ 0. \\ 0. \\ -0.000703 \\ -16000. \\ 0. \end{bmatrix} \quad (10)$$

$(UR_2 + UR_8) \cdot 1.0$

$$-80000.00000 \quad (11)$$

Fra FEM-Design

Point support group, reactions, Load comb.: Lodret

ID	x	y	z	Node	Fx'	Fy'	Fz'	Mx'	My'	Mz'
[-]	[m]	[m]	[m]	[-]	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
S.1	0.000	0.000	0.000	1	0.000	0.000	-64.027	0.000	42.775	0.000
S.3	4.000	0.000	-3.000	3	0.000	0.000	-15.973	0.000	0.000	0.000

Taleksempel.

Element 1 er HEA160 og element 2 er HEA160 med momentstiv samling.

$I1 := 16700000; E := 210000; L1 := 4000; A1 := 3877; L2 := 3000; q := 40; I2 := I1; A2 := A1$

$$16700000$$
$$210000$$
$$4000$$
$$3877$$
$$3000$$
$$40$$
$$16700000$$
$$3877 \quad (12)$$

$\frac{E \cdot A2}{L2}$

$$271390 \quad (13)$$

$UR := U - Uv :$

$evalf_3(UR \cdot 1.0)$

$$\begin{bmatrix} 3540. \\ -60000. \\ 3.74\,10^7 \\ 0. \\ 0. \\ 0. \\ -3540. \\ -20000. \\ 0. \end{bmatrix} \quad (14)$$

$(UR_2 + UR_8)$

$$-80000 \quad (15)$$

Fra FEM-Design

Point support group, reactions, Load comb.: Lodret

ID	x	y	z	Node	Fx'	Fy'	Fz'	Mx'	My'	Mz'
[-]	[m]	[m]	[m]	[-]	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
S.1	0.000	0.000	0.000	1	-3.538	0.000	-60.029	0.000	37.396	0.000
S.3	4.000	0.000	-3.000	3	3.538	0.000	-19.971	0.000	0.000	0.000

Kontrol af stivhedsmatrice

K =

Columns 1 through 7

2,04E+08	0	0	-2,04E+08	0	0	0	0	0
0	657.6	1,32E+09	0	-657.6	1,32E+09	0	0	0
0	1,32E+09	3,51E+12	0	-1,32E+09	1,75E+12	0	0	0
-2,04E+08	0	0	2,05E+08	0	2,34E+09	-1559	0	2,34E+09
0	-657.6	-1,32E+09	0	2.72e+05	-1,32E+09	0	-2,71E+08	0
0	1,32E+09	1,75E+12	2,34E+09	-1,32E+09	8,18E+12	-2,34E+09	0	2,34E+12
0	0	0	-1559	0	-2,34E+09	1559	0	-2,34E+09
0	0	0	0	-2,71E+08	0	0	2,71E+08	0
0	0	0	2,34E+09	0	2,34E+12	-2,34E+09	0	4,68E+12

$evalf_3(K \cdot 1000.0)$

$$\begin{bmatrix} 2.04\,10^{8} & 0. & 0. & -2.04\,10^{8} & 0. & 0. & 0. & 0. & 0. \\ 0. & 6.58\,10^{5} & -1.32\,10^{9} & 0. & -6.58\,10^{5} & -1.32\,10^{9} & 0. & 0. & 0. \\ 0. & -1.32\,10^{9} & 3.51\,10^{12} & 0. & 1.32\,10^{9} & 1.75\,10^{12} & 0. & 0. & 0. \\ -2.04\,10^{8} & 0. & 0. & 2.05\,10^{8} & 0. & 2.34\,10^{9} & -1.56\,10^{6} & 0. & 2.34\,10^{9} \\ 0. & -6.58\,10^{5} & 1.32\,10^{9} & 0. & 2.72\,10^{8} & 1.32\,10^{9} & 0. & -2.71\,10^{8} & 0. \\ 0. & -1.32\,10^{9} & 1.75\,10^{12} & 2.34\,10^{9} & 1.32\,10^{9} & 8.18\,10^{12} & -2.34\,10^{9} & 0. & 2.34\,10^{12} \\ 0. & 0. & 0. & -1.56\,10^{6} & 0. & -2.34\,10^{9} & 1.56\,10^{6} & 0. & -2.34\,10^{9} \\ 0. & 0. & 0. & 0. & -2.71\,10^{8} & 0. & 0. & 2.71\,10^{8} & 0. \\ 0. & 0. & 0. & 2.34\,10^{9} & 0. & 2.34\,10^{12} & -2.34\,10^{9} & 0. & 4.68\,10^{12} \end{bmatrix} \tag{16}$$