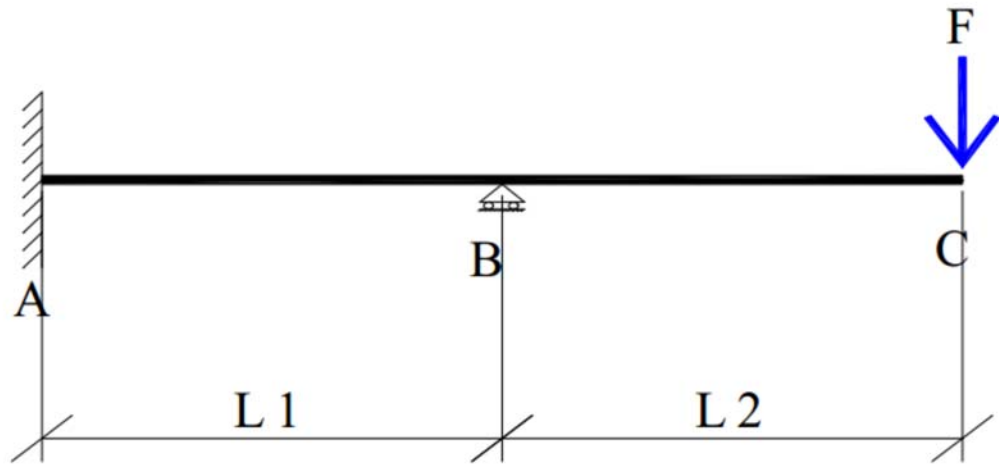


Løsningsforslag til opgave 7.4.1



Bjælken er HE 160A . Længden L1 er 5 m og længden L2 er 4 m.

F er 10 kN..

with(Student[LinearAlgebra]) :
restart;

$$Ke1 := Es \cdot Iy \cdot \begin{bmatrix} \frac{12}{L1^3} & -\frac{6}{L1^2} & -\frac{12}{L1^3} & -\frac{6}{L1^2} \\ -\frac{6}{L1^2} & \frac{4}{L1} & \frac{6}{L1^2} & \frac{2}{L1} \\ -\frac{12}{L1^3} & \frac{6}{L1^2} & \frac{12}{L1^3} & \frac{6}{L1^2} \\ -\frac{6}{L1^2} & \frac{2}{L1} & \frac{6}{L1^2} & \frac{4}{L1} \end{bmatrix}$$

$$\begin{bmatrix} \frac{12 Es Iy}{L1^3} & -\frac{6 Es Iy}{L1^2} & -\frac{12 Es Iy}{L1^3} & -\frac{6 Es Iy}{L1^2} \\ -\frac{6 Es Iy}{L1^2} & \frac{4 Es Iy}{L1} & \frac{6 Es Iy}{L1^2} & \frac{2 Es Iy}{L1} \\ -\frac{12 Es Iy}{L1^3} & \frac{6 Es Iy}{L1^2} & \frac{12 Es Iy}{L1^3} & \frac{6 Es Iy}{L1^2} \\ -\frac{6 Es Iy}{L1^2} & \frac{2 Es Iy}{L1} & \frac{6 Es Iy}{L1^2} & \frac{4 Es Iy}{L1} \end{bmatrix}$$

(1)

$$\begin{aligned}
 Ke2 &:= Es \cdot Iy \cdot \begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} \\
 & \begin{bmatrix} \frac{12 Es Iy}{L^3} & -\frac{6 Es Iy}{L^2} & -\frac{12 Es Iy}{L^3} & -\frac{6 Es Iy}{L^2} \\ -\frac{6 Es Iy}{L^2} & \frac{4 Es Iy}{L} & \frac{6 Es Iy}{L^2} & \frac{2 Es Iy}{L} \\ -\frac{12 Es Iy}{L^3} & \frac{6 Es Iy}{L^2} & \frac{12 Es Iy}{L^3} & \frac{6 Es Iy}{L^2} \\ -\frac{6 Es Iy}{L^2} & \frac{2 Es Iy}{L} & \frac{6 Es Iy}{L^2} & \frac{4 Es Iy}{L} \end{bmatrix} \tag{2}
 \end{aligned}$$

$$K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$K1 := K11 \cdot Ke1 \cdot K11^+$$

$$\begin{bmatrix}
 \frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & 0 & 0 \\
 -\frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & 0 & 0 \\
 -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & 0 & 0 \\
 -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix} \quad (4)$$

$$K21 := \begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
 \end{bmatrix} \quad (5)$$

$$K2 := K21 \cdot Ke2 \cdot K21^+$$

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{12 Es Iy}{L2^3} & -\frac{6 Es Iy}{L2^2} & -\frac{12 Es Iy}{L2^3} & -\frac{6 Es Iy}{L2^2} \\
 0 & 0 & -\frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} \\
 0 & 0 & -\frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} & \frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} \\
 0 & 0 & -\frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2}
 \end{bmatrix} \quad (6)$$

$$K := K1 + K2$$

$$\begin{aligned}
& \left[\left[\frac{12 Es Iy}{L1^3}, -\frac{6 Es Iy}{L1^2}, -\frac{12 Es Iy}{L1^3}, -\frac{6 Es Iy}{L1^2}, 0, 0 \right], \right. \\
& \left[-\frac{6 Es Iy}{L1^2}, \frac{4 Es Iy}{L1}, \frac{6 Es Iy}{L1^2}, \frac{2 Es Iy}{L1}, 0, 0 \right], \\
& \left[-\frac{12 Es Iy}{L1^3}, \frac{6 Es Iy}{L1^2}, \frac{12 Es Iy}{L1^3} + \frac{12 Es Iy}{L2^3}, \frac{6 Es Iy}{L1^2} - \frac{6 Es Iy}{L2^2}, -\frac{12 Es Iy}{L2^3}, -\frac{6 Es Iy}{L2^2} \right. \\
& \left. \left[-\frac{6 Es Iy}{L1^2}, \frac{2 Es Iy}{L1}, \frac{6 Es Iy}{L1^2} - \frac{6 Es Iy}{L2^2}, \frac{4 Es Iy}{L1} + \frac{4 Es Iy}{L2}, \frac{6 Es Iy}{L2^2}, \frac{2 Es Iy}{L2} \right] \right. \\
& \left. \left[0, 0, -\frac{12 Es Iy}{L2^3}, \frac{6 Es Iy}{L2^2}, \frac{12 Es Iy}{L2^3}, \frac{6 Es Iy}{L2^2} \right], \right. \\
& \left. \left[0, 0, -\frac{6 Es Iy}{L2^2}, \frac{2 Es Iy}{L2}, \frac{6 Es Iy}{L2^2}, \frac{4 Es Iy}{L2} \right] \right]
\end{aligned} \tag{7}$$

$$p11 := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(8)

$$Kmod := p11^+ . K . p11$$

$$\begin{bmatrix} \frac{4 Es Iy}{L1} + \frac{4 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} \\ \frac{6 Es Iy}{L2^2} & \frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} \\ \frac{2 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2} \end{bmatrix}$$

(9)

$$Umod := \begin{bmatrix} 0 \\ F \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ F \\ 0 \end{bmatrix} \quad (10)$$

$$u_{mod} := K_{mod}^{-1} \cdot U_{mod}$$

$$\begin{bmatrix} -\frac{1}{4} \frac{L2 L1 F}{Es Iy} \\ \frac{1}{12} \frac{(3 L1 + 4 L2) L2^2 F}{Es Iy} \\ -\frac{1}{4} \frac{(L1 + 2 L2) L2 F}{Es Iy} \end{bmatrix} \quad (11)$$

$$pu := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

$$u := pu \cdot u_{mod}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{4} \frac{L2 L1 F}{Es Iy} \\ \frac{1}{12} \frac{(3 L1 + 4 L2) L2^2 F}{Es Iy} \\ -\frac{1}{4} \frac{(L1 + 2 L2) L2 F}{Es Iy} \end{bmatrix} \quad (13)$$

$$U := simplify(K \cdot u)$$

$$\begin{bmatrix} \frac{3}{2} \frac{L2 F}{LI} \\ -\frac{1}{2} L2 F \\ -\frac{1}{2} \frac{F (3 L2 + 2 LI)}{LI} \\ 0 \\ F \\ 0 \end{bmatrix} \quad (14)$$

$$F := 10 \cdot kN; I_y := 16.73 \cdot 10^6 \cdot mm^4; LI := 5 \cdot m; L2 := 4 \cdot m; Es := \frac{210000 \cdot N}{mm^2}$$

$$\begin{bmatrix} 10 \text{ kN} \\ 1.673000000 \cdot 10^7 \text{ mm}^4 \\ 5 \text{ m} \\ 4 \text{ m} \\ \frac{210000 \text{ N}}{mm^2} \\ 0 \end{bmatrix} \quad (15)$$

K.u

$$\begin{bmatrix} 12,0 \text{ kN} \\ -20,0 \text{ m kN} \\ -22,0 \text{ kN} \\ 0,0 \\ 10,0 \text{ kN} \\ 0,0 \end{bmatrix} \quad (16)$$