BFEM. Opgave 7.4.2. Løsningsforslag.

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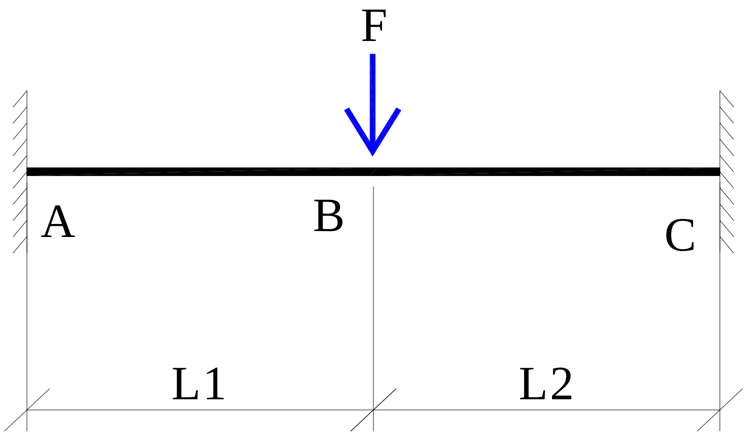
Opgave 10, FEM-beregning af bjælke, tvangsdeformation

Fortegnsdefinition som FEM notatet.



Bjælken er HE 160A . Længden L1 er 5 m og længden L2 er 4 m.

Pkt. B skal have en deformation på 20 mm.

A) Find den nødvendige kraft F.

B) Find reaktionerne i A og i C samt vinkeldrejning i B.

C) Find momentet i B.

D) Find stivhedsmatrice og snitkræfter, hvis der indføres charnier i B.

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Løsningsforslag

$L1 := 5000$ $L2 := 4000$ $u_b := 20$ $E := 210000$

$E1 := 210000$ $E2 := 210000$ $I1 := 16.7\cdot 10^6$ $I2 := 16.7\cdot 10^6$

$$K1 := \frac{E1\cdot I1}{L1^2}\cdot\begin{pmatrix} \frac{12}{L1} & -6 & -\frac{12}{L1} & -6 & 0 & 0 \\ -6 & 4\cdot L1 & 6 & 2\cdot L1 & 0 & 0 \\ -\frac{12}{L1} & 6 & \frac{12}{L1} & 6 & 0 & 0 \\ -6 & 2\cdot L1 & 6 & 4\cdot L1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K2 := \frac{E2\cdot I2}{L2^2}\cdot\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12}{L2} & -6 & \frac{-12}{L2} & -6 \\ 0 & 0 & -6 & 4\cdot L2 & 6 & 2\cdot L2 \\ 0 & 0 & \frac{-12}{L2} & 6 & \frac{12}{L2} & 6 \\ 0 & 0 & -6 & 2\cdot L2 & 6 & 4\cdot L2 \end{pmatrix}$$

$$u = \begin{pmatrix} u_a \\ r_a \\ u_b \\ r_b \\ u_c \\ r_c \end{pmatrix} \quad u = \begin{pmatrix} 0 \\ 0 \\ u_b \\ r_b \\ 0 \\ 0 \end{pmatrix} \quad U_b = \begin{pmatrix} R_a \\ M_a \\ R_b \\ M_b \\ R_c \\ M_c \end{pmatrix} \quad U_b = \begin{pmatrix} R_a \\ M_a \\ 0 \\ 0 \\ R_c \\ M_c \end{pmatrix} \quad U_l = \begin{pmatrix} 0 \\ 0 \\ F_b \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$K := K1 + K2 \qquad K_{mod} := \begin{pmatrix} K_{3,3} & K_{3,4} \\ K_{4,3} & K_{4,4} \end{pmatrix} \qquad K_{mod}\cdot\begin{pmatrix} u_b \\ r_b \end{pmatrix} = \begin{pmatrix} F_b \\ 0 \end{pmatrix}$$

$$K_{3,3}\cdot u_b + K_{3,4}\cdot r_b = F_b \qquad K_{4,3}\cdot u_b + K_{4,4}\cdot r_b = 0$$

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$$r_b := \frac{-(K_{4,3} \cdot u_b)}{K_{4,4}} \qquad r_b = 1.5 \times 10^{-3}$$

$$F_b := K_{3,3} \cdot u_b + K_{3,4} \cdot \frac{-(K_{4,3} \cdot u_b)}{K_{4,4}} \qquad F_b = 1.917 \times 10^{4}$$

$$\begin{pmatrix} R_a \\ M_a \\ F_b \\ M_b \\ R_c \\ M_c \end{pmatrix} = K \cdot \begin{pmatrix} 0 \\ 0 \\ u_b \\ r_b \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} R_a \\ 1 \\ 19174.522500000000000 \\ 0 \\ R_c \\ M_c \end{pmatrix} = \begin{pmatrix} -7995.9600000000000000 \\ 18937800.000000000000 \\ 19174.522500000000000 \\ 0 \\ -11178.562500000000000 \\ -23672250.000000000000 \end{pmatrix}$$

Moment i B:

$$M_b := 7995.96 \cdot L1 - 18937800.0 \rightarrow 21042000.00$$

$$K1 := \frac{E1 \cdot I1}{L1^2} \cdot \begin{pmatrix} \frac{12}{L1} & -6 & \frac{-12}{L1} & -6 \\ -6 & 4 \cdot L1 & 6 & 2 \cdot L1 \\ \frac{-12}{L1} & 6 & \frac{12}{L1} & 6 \\ -6 & 2 \cdot L1 & 6 & 4 \cdot L1 \end{pmatrix} \qquad u := \begin{pmatrix} 0 \\ 0 \\ u_b \\ r_b \end{pmatrix} \qquad U_b = \begin{pmatrix} R_a \\ M_a \\ V_b \\ M_b \end{pmatrix}$$

$$\begin{pmatrix} R_a \\ M_a \\ V_b \\ M_b \end{pmatrix} := K1 \cdot u \rightarrow \begin{pmatrix} -7995.9600000000000000 \\ 18937800.000000000000 \\ 7995.9600000000000000 \\ 21042000.000000000000 \end{pmatrix}$$

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Charnier i B. Opdeling i 2 separate bjælker:

Bjælke 1

$$K1 := \frac{E1 \cdot I1}{L1^2} \cdot \begin{pmatrix} \frac{12}{L1} & -6 & \frac{-12}{L1} & -6 \\ -6 & 4 \cdot L1 & 6 & 2 \cdot L1 \\ \frac{-12}{L1} & 6 & \frac{12}{L1} & 6 \\ -6 & 2 \cdot L1 & 6 & 4 \cdot L1 \end{pmatrix} \qquad U_b = \begin{pmatrix} R_a \\ M_a \\ 0 \\ 0 \end{pmatrix} \qquad U_l := \begin{pmatrix} 0 \\ 0 \\ V_b \\ 0 \end{pmatrix} \qquad u_b = 20$$

$$r_b := -\frac{K1_{4,3} \cdot u_b}{K1_{4,4}} \qquad r_b = -6 \times 10^{-3} \qquad V_b := K1_{3,3} \cdot u_b + K1_{3,4} \cdot r_b \qquad V_b = 1.683 \times 10^{3}$$

$$u := \begin{pmatrix} 0 \\ 0 \\ u_b \\ r_b \end{pmatrix} \qquad K1 \cdot u = \begin{pmatrix} -1.683 \times 10^{3} \\ 8.417 \times 10^{6} \\ 1.683 \times 10^{3} \\ -3.511 \times 10^{-10} \end{pmatrix}$$

Bjælke 2

$$K2 := \frac{E2 \cdot I2}{L2^2} \cdot \begin{pmatrix} \frac{12}{L2} & -6 & \frac{-12}{L2} & -6 \\ -6 & 4 \cdot L2 & 6 & 2 \cdot L2 \\ \frac{-12}{L2} & 6 & \frac{12}{L2} & 6 \\ -6 & 2 \cdot L2 & 6 & 4 \cdot L2 \end{pmatrix} \qquad U_b = \begin{pmatrix} 0 \\ 0 \\ R_c \\ M_c \end{pmatrix} \qquad U_l := \begin{pmatrix} V_b \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$r_b := -\frac{K2_{2,1} \cdot u_b}{K2_{2,2}} \qquad r_b = 7.5 \times 10^{-3} \qquad V_b := K2_{1,1} \cdot u_b + K2_{1,2} \cdot r_b \qquad V_b = 3.288 \times 10^{3}$$

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$$u := \begin{pmatrix} u_b \\ r_b \\ 0 \\ 0 \end{pmatrix} \qquad K2 \cdot u = \begin{pmatrix} 3.288 \times 10^{3} \\ -9.732 \times 10^{-10} \\ -3.288 \times 10^{3} \\ -1.315 \times 10^{7} \end{pmatrix}$$

Samlet kraft i B: $1.68 \cdot 10^{3} + 3.29 \cdot 10^{3} = 4.97 \times 10^{3}$

Samlet stivhedsmatrice for bjælken med charnier

$$K1 := \frac{E1 \cdot I1}{L1^{2}} \cdot \begin{pmatrix} \frac{12}{L1} & -6 & -\frac{12}{L1} & -6 & 0 & 0 & 0 \\ -6 & 4 \cdot L1 & 6 & 2 \cdot L1 & 0 & 0 & 0 \\ -\frac{12}{L1} & 6 & \frac{12}{L1} & 6 & 0 & 0 & 0 \\ -6 & 2 \cdot L1 & 6 & 4 \cdot L1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad K2 := \frac{E2 \cdot I2}{L2^{2}} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12}{L2} & 0 & -6 & \frac{-12}{L2} & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 & 4 \cdot L2 & 6 & 2 \cdot L2 \\ 0 & 0 & \frac{-12}{L2} & 0 & 6 & \frac{12}{L2} & 6 \\ 0 & 0 & -6 & 0 & 2 \cdot L2 & 6 & 4 \cdot L2 \end{pmatrix}$$

$K := K1 + K2$

$$u = \begin{pmatrix} 0 \\ 0 \\ u_b \\ r_{Bv} \\ r_{Bh} \\ 0 \\ 0 \end{pmatrix} \qquad U = \begin{pmatrix} R_A \\ M_A \\ F1 \\ 0 \\ 0 \\ R_C \\ M_C \end{pmatrix}$$

$i := 1 .. 3 \qquad j := 1 .. 3$

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$K_{mod_{i,j}} := K_{i+2,j+2}$

$$K_{mod} = \begin{pmatrix} 994.235 & 8.417\times10^{5} & -1.315\times10^{6} \\ 8.417\times10^{5} & 2.806\times10^{9} & 0 \\ -1.315\times10^{6} & 0 & 3.507\times10^{9} \end{pmatrix}$$

$F1 := 1$ $\quad r_{Bv} := 1$ $\quad r_{Bh} := 1$

Given

$$K_{mod_{1,1}}\cdot u_b + K_{mod_{1,2}}\cdot r_{Bv} + K_{mod_{1,3}}\cdot r_{Bh} = F1$$

$$K_{mod_{2,1}}\cdot u_b + K_{mod_{2,2}}\cdot r_{Bv} + K_{mod_{2,3}}\cdot r_{Bh} = 0$$

$$K_{mod_{3,1}}\cdot u_b + K_{mod_{3,2}}\cdot r_{Bv} + K_{mod_{3,3}}\cdot r_{Bh} = 0$$

$f := \text{Find}(F1, r_{Bv}, r_{Bh})$

$$f = \begin{pmatrix} 4.971\times10^{3} \\ -6\times10^{-3} \\ 7.5\times10^{-3} \end{pmatrix}$$

$$u := \begin{pmatrix} 0 \\ 0 \\ u_b \\ f_2 \\ f_3 \\ 0 \\ 0 \end{pmatrix} \qquad K\cdot u = \begin{pmatrix} -1.683\times10^{3} \\ 8.417\times10^{6} \\ 4.971\times10^{3} \\ -3.511\times10^{-10} \\ -6.222\times10^{-6} \\ -3.288\times10^{3} \\ -1.315\times10^{7} \end{pmatrix}$$