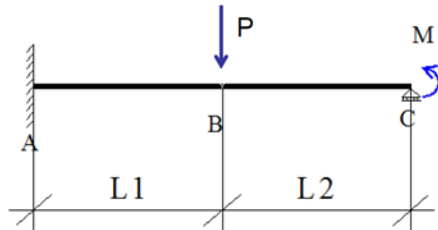


> with(Student[LinearAlgebra]) :

Løsningsforslag til opgave 9.4.1

Opgave 9.4.1.



Der er charnier i punkt B.

- A) Find reaktioner.
- B) Optegn snitkraftkurver for forskydning og moment.
- C) Kontroller resultatet med et FEM software.
- D) Opgave 7.4.2 med charnier i knude B.

> restart;

$$\text{> } Kel := Es \cdot Iy \cdot \begin{bmatrix} \frac{12}{Ll^3} & -\frac{6}{Ll^2} & -\frac{12}{Ll^3} & -\frac{6}{Ll^2} \\ -\frac{6}{Ll^2} & \frac{4}{Ll} & \frac{6}{Ll^2} & \frac{2}{Ll} \\ -\frac{12}{Ll^3} & \frac{6}{Ll^2} & \frac{12}{Ll^3} & \frac{6}{Ll^2} \\ -\frac{6}{Ll^2} & \frac{2}{Ll} & \frac{6}{Ll^2} & \frac{4}{Ll} \end{bmatrix}$$

$$Ke1 := \begin{bmatrix} \frac{12 Es Iy}{L1^3} & -\frac{6 Es Iy}{L1^2} & -\frac{12 Es Iy}{L1^3} & -\frac{6 Es Iy}{L1^2} \\ -\frac{6 Es Iy}{L1^2} & \frac{4 Es Iy}{L1} & \frac{6 Es Iy}{L1^2} & \frac{2 Es Iy}{L1} \\ -\frac{12 Es Iy}{L1^3} & \frac{6 Es Iy}{L1^2} & \frac{12 Es Iy}{L1^3} & \frac{6 Es Iy}{L1^2} \\ -\frac{6 Es Iy}{L1^2} & \frac{2 Es Iy}{L1} & \frac{6 Es Iy}{L1^2} & \frac{4 Es Iy}{L1} \end{bmatrix} \quad (1)$$

$$> Ke2 := Es \cdot Iy \cdot \begin{bmatrix} \frac{12}{L2^3} & -\frac{6}{L2^2} & -\frac{12}{L2^3} & -\frac{6}{L2^2} \\ -\frac{6}{L2^2} & \frac{4}{L2} & \frac{6}{L2^2} & \frac{2}{L2} \\ -\frac{12}{L2^3} & \frac{6}{L2^2} & \frac{12}{L2^3} & \frac{6}{L2^2} \\ -\frac{6}{L2^2} & \frac{2}{L2} & \frac{6}{L2^2} & \frac{4}{L2} \end{bmatrix}$$

$$Ke2 := \begin{bmatrix} \frac{12 Es Iy}{L2^3} & -\frac{6 Es Iy}{L2^2} & -\frac{12 Es Iy}{L2^3} & -\frac{6 Es Iy}{L2^2} \\ -\frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} \\ -\frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} & \frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} \\ -\frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2} \end{bmatrix} \quad (2)$$

$$> K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K11 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

> LinearAlgebra:-Transpose((3))

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

> K12 := LinearAlgebra:-Transpose(K11)

$$K12 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

> K1 := K11.Kel.K12

$$K1 := \begin{bmatrix} \frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & -\frac{12 Es Iy}{LI^3} & -\frac{6 Es Iy}{LI^2} & 0 & 0 & 0 \\ -\frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & 0 & 0 & 0 \\ -\frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & \frac{12 Es Iy}{LI^3} & \frac{6 Es Iy}{LI^2} & 0 & 0 & 0 \\ -\frac{6 Es Iy}{LI^2} & \frac{2 Es Iy}{LI} & \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

>

$$\text{> } K21 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K21 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$\text{> } K22 := \text{LinearAlgebra:-Transpose}(K21)$$

$$K22 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$\text{> } K2 := K21.K2.K22$$

$$K2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12 Es Iy}{L2^3} & 0 & -\frac{6 Es Iy}{L2^2} & -\frac{12 Es Iy}{L2^3} & -\frac{6 Es Iy}{L2^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{6 Es Iy}{L2^2} & 0 & \frac{4 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{2 Es Iy}{L2} \\ 0 & 0 & -\frac{12 Es Iy}{L2^3} & 0 & \frac{6 Es Iy}{L2^2} & \frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{L2^2} \\ 0 & 0 & -\frac{6 Es Iy}{L2^2} & 0 & \frac{2 Es Iy}{L2} & \frac{6 Es Iy}{L2^2} & \frac{4 Es Iy}{L2} \end{bmatrix} \quad (9)$$

$$\text{> } K := K1 + K2$$

$$K := \left[\left[\frac{12 Es Iy}{L1^3}, -\frac{6 Es Iy}{L1^2}, -\frac{12 Es Iy}{L1^3}, -\frac{6 Es Iy}{L1^2}, 0, 0, 0 \right], \right] \quad (10)$$

$$\begin{aligned} & \left[-\frac{6 Es Iy}{Ll^2}, \frac{4 Es Iy}{Ll}, \frac{6 Es Iy}{Ll^2}, \frac{2 Es Iy}{Ll}, 0, 0, 0 \right], \\ & \left[-\frac{12 Es Iy}{Ll^3}, \frac{6 Es Iy}{Ll^2}, \frac{12 Es Iy}{Ll^3} + \frac{12 Es Iy}{L2^3}, \frac{6 Es Iy}{Ll^2}, -\frac{6 Es Iy}{L2^2}, -\frac{12 Es Iy}{L2^3}, -\frac{6 Es Iy}{L2^2} \right], \\ & \left[-\frac{6 Es Iy}{Ll^2}, \frac{2 Es Iy}{Ll}, \frac{6 Es Iy}{Ll^2}, \frac{4 Es Iy}{Ll}, 0, 0, 0 \right], \\ & \left[0, 0, -\frac{6 Es Iy}{L2^2}, 0, \frac{4 Es Iy}{L2}, \frac{6 Es Iy}{L2^2}, \frac{2 Es Iy}{L2} \right], \\ & \left[0, 0, -\frac{12 Es Iy}{L2^3}, 0, \frac{6 Es Iy}{L2^2}, \frac{12 Es Iy}{L2^3}, \frac{6 Es Iy}{L2^2} \right], \\ & \left[0, 0, -\frac{6 Es Iy}{L2^2}, 0, \frac{2 Es Iy}{L2}, \frac{6 Es Iy}{L2^2}, \frac{4 Es Iy}{L2} \right] \end{aligned}$$

$$\text{> } p11 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p11 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(11)

$$\text{> } p12 := \text{LinearAlgebra:-Transpose}(p11)$$

$$p12 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(12)

$$\text{> } Kmod := p12.K.p11$$

$$K_{mod} := \begin{bmatrix} \frac{12 Es Iy}{LI^3} + \frac{12 Es Iy}{L2^3} & \frac{6 Es Iy}{LI^2} & -\frac{6 Es Iy}{L2^2} & -\frac{6 Es Iy}{L2^2} \\ \frac{6 Es Iy}{LI^2} & \frac{4 Es Iy}{LI} & 0 & 0 \\ -\frac{6 Es Iy}{L2^2} & 0 & \frac{4 Es Iy}{L2} & \frac{2 Es Iy}{L2} \\ -\frac{6 Es Iy}{L2^2} & 0 & \frac{2 Es Iy}{L2} & \frac{4 Es Iy}{L2} \end{bmatrix} \quad (13)$$

$$> U_{mod} := \begin{bmatrix} Pb \\ 0 \\ 0 \\ M \end{bmatrix}$$

$$U_{mod} := \begin{bmatrix} Pb \\ 0 \\ 0 \\ M \end{bmatrix} \quad (14)$$

$$> u_{mod} := K_{mod}^{-1} \cdot U_{mod}$$

$$u_{mod} := \begin{bmatrix} \frac{1}{3} \frac{LI^3 Pb}{Es Iy} + \frac{1}{3} \frac{LI^3 M}{L2 Es Iy} \\ -\frac{1}{2} \frac{LI^2 Pb}{Es Iy} - \frac{1}{2} \frac{LI^2 M}{L2 Es Iy} \\ \frac{1}{3} \frac{LI^3 Pb}{L2 Es Iy} + \frac{1}{6} \frac{(2 LI^3 - L2^3) M}{Es Iy L2^2} \\ \frac{1}{3} \frac{LI^3 Pb}{L2 Es Iy} + \frac{1}{3} \frac{(L2^3 + LI^3) M}{Es Iy L2^2} \end{bmatrix} \quad (15)$$



$$\begin{aligned} I_y &:= 57000000 \text{ mm}^4 \\ L1 &:= 4 \text{ m} \\ L2 &:= 5 \text{ m} \\ E_s &:= \frac{210000 \text{ N}}{\text{mm}^2} \end{aligned} \tag{18}$$

$$\begin{bmatrix} -12 \text{ kN} \\ 48 \text{ kN m} \\ 10 \text{ kN} \\ 0 \\ 0 \\ 2 \text{ kN} \\ 10 \text{ kN m} \end{bmatrix} \tag{19}$$