BFEM. Opgave 2.4.1. Løsningsforslag

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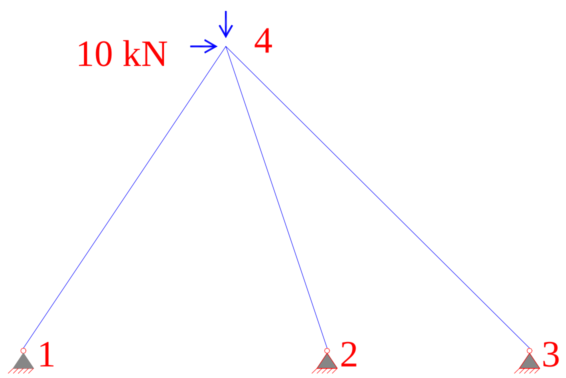
FEM opgave, gitterkonstruktion.



Ovenfor er der vist en gitterkonstruktion. Alle 3 stænger er cirkulære rør i stål S235 med udvendig diameter på 120 mm og godstykkelse på 10 mm.

Vandret last i knude 4 er 10 kN og lodret last i knude 4 er 15 kN i det angivne koordinatsystem.

A) Opstil stivhedsmatrice for de enkelte elementer i lokale og globale koordinater.
Knude 1 har koordinater i meter: (0,0), knude 2: (3,0), knude 3: (5,0) og knude 4: (2,3)

B) Opstil stivhedsmatrix for det samlede system.

C) Løs ligningssystemet og find deformationerne i knude 4.

D) Find stangkræfterne i alle 3 stænger.

E) Kontrollér resultaterne med EDB. F. eks. Analys.

Tip:

Deformationer i det globale koordinatsystem kan benævnes således:

Vandret - henholdsvis lodret deformation i knude 1 kaldes p1 henholdsvis p2.
Vandret - henholdsvis lodret deformation i knude 2 kaldes p3 henholdsvis p4
Vandret - henholdsvis lodret deformation i knude 3 kaldes p5 henholdsvis p6
Vandret - henholdsvis lodret deformation i knude 4 kaldes p7 henholdsvis p8

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Løsningsforslag.

$$T^T \cdot K^e \cdot T = \begin{pmatrix} \cos(\gamma) & 0 \\ \sin(\gamma) & 0 \\ 0 & \cos(\gamma) \\ 0 & \sin(\gamma) \end{pmatrix} \cdot \frac{E \cdot A}{L} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\gamma) & \sin(\gamma) & 0 & 0 \\ 0 & 0 & \cos(\gamma) & \sin(\gamma) \end{pmatrix}$$

Element 1: $\gamma1 := -\mathrm{atan}\left(\frac{3}{2}\right)$ $\gamma1 = -0.983$ $E := 210000000$

$L1 := \sqrt{3^2 + 2^2}$ $L1 = 3.606$ $A := \frac{\pi}{4} \cdot \left(120^2 - 100^2\right) \cdot 10^{-6}$ $A = 3.456 \times 10^{-3}$

$$K1 := \begin{pmatrix} \cos(\gamma1) & 0 \\ \sin(\gamma1) & 0 \\ 0 & \cos(\gamma1) \\ 0 & \sin(\gamma1) \end{pmatrix} \cdot \frac{E \cdot A}{L1} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\gamma1) & \sin(\gamma1) & 0 & 0 \\ 0 & 0 & \cos(\gamma1) & \sin(\gamma1) \end{pmatrix}$$

Element 2: $\gamma2 := \mathrm{atan}\left(\frac{3}{1}\right)$ $\gamma2 = 1.249$ $E := 210000000$

$L2 := \sqrt{3^2 + 1^2}$ $L2 = 3.162$ $A := \frac{\pi}{4} \cdot \left(120^2 - 100^2\right) \cdot 10^{-6}$ $A = 3.456 \times 10^{-3}$

$$K2 := \begin{pmatrix} \cos(\gamma2) & 0 \\ \sin(\gamma2) & 0 \\ 0 & \cos(\gamma2) \\ 0 & \sin(\gamma2) \end{pmatrix} \cdot \frac{E \cdot A}{L2} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\gamma2) & \sin(\gamma2) & 0 & 0 \\ 0 & 0 & \cos(\gamma2) & \sin(\gamma2) \end{pmatrix}$$

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Element 3: $\gamma3 := \operatorname{atan}\left(\frac{3}{3}\right)$ $\gamma3 = 0.785$ $E := 210000000$ $A := \frac{\pi}{4}\cdot\left(120^2 - 100^2\right)\cdot 10^{-6}$

$L3 := \sqrt{3^2 + 3^2}$ $L3 = 4.243$ $A = 3.456 \times 10^{-3}$

$$K3 := \begin{pmatrix} \cos(\gamma3) & 0 \\ \sin(\gamma3) & 0 \\ 0 & \cos(\gamma3) \\ 0 & \sin(\gamma3) \end{pmatrix}\cdot\frac{E\cdot A}{L3}\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\gamma3) & \sin(\gamma3) & 0 & 0 \\ 0 & 0 & \cos(\gamma3) & \sin(\gamma3) \end{pmatrix}$$

K nulstilles:

$i := 1..8$ $j := 1..8$ $K_{i,j} := 0$

Element 1 har følgende pladser i K:

$$\begin{pmatrix} K_{1,1} & K_{1,2} & K_{1,7} & K_{1,8} \\ K_{2,1} & K_{2,2} & K_{2,7} & K_{2,8} \\ K_{7,1} & K_{7,2} & K_{7,7} & K_{7,8} \\ K_{8,1} & K_{8,2} & K_{8,7} & K_{8,8} \end{pmatrix}$$

$K_{1,1} := K1_{1,1}$ $K_{1,2} := K1_{1,2}$

$K_{1,7} := K1_{1,3}$ $K_{1,8} := K1_{1,4}$

$K_{2,7} := K1_{2,3}$ $K_{2,8} := K1_{2,4}$

$K_{2,2} := K1_{2,2}$

Element 2 har følgende pladser i K:

$$\begin{pmatrix} K_{7,7} & K_{7,8} & K_{7,3} & K_{7,4} \\ K_{8,7} & K_{8,8} & K_{8,3} & K_{8,4} \\ K_{3,7} & K_{3,8} & K_{3,3} & K_{3,4} \\ K_{4,7} & K_{4,8} & K_{4,3} & K_{4,4} \end{pmatrix}$$

$K_{3,3} := K2_{3,3}$ $K_{4,4} := K2_{4,4}$

$K_{3,4} := K2_{3,4}$

$K_{3,7} := K2_{3,1}$ $K_{4,7} := K2_{4,1}$

$K_{3,8} := K2_{3,2}$ $K_{4,8} := K2_{4,2}$

Element 3 har følgende pladser i K:

$$\begin{pmatrix} K_{7,7} & K_{7,8} & K_{7,5} & K_{7,6} \\ K_{8,7} & K_{8,8} & K_{8,5} & K_{8,6} \\ K_{5,7} & K_{5,8} & K_{5,5} & K_{5,6} \\ K_{6,7} & K_{6,8} & K_{6,5} & K_{6,6} \end{pmatrix}$$

$K_{5,5} := K3_{3,3}$ $K_{6,6} := K3_{4,4}$

$K_{5,6} := K3_{3,4}$

$K_{5,7} := K3_{3,1}$ $K_{5,8} := K3_{3,2}$

$K_{6,7} := K3_{4,1}$ $K_{6,8} := K3_{4,2}$

$K_{7,7} := K1_{3,3} + K2_{1,1} + K3_{1,1}$ $K_{8,8} := K1_{4,4} + K2_{2,2} + K3_{2,2}$ $K_{7,8} := K1_{3,4} + K2_{1,2} + K3_{1,2}$

$K_{7,8} = 6.148 \times 10^{4}$

Det udnyttes, at K er symmetrisk:

$Km := K + K^{T}$ $K := Km$ $i := 1..8$ $K_{i,i} := \frac{Km_{i,i}}{2}$

$$K = \begin{pmatrix} 6.193 \times 10^{4} & -9.29 \times 10^{4} & 0 & 0 & 0 & 0 & -6.193 \times 10^{4} & 9.29 \times 10^{4} \\ -9.29 \times 10^{4} & 1.393 \times 10^{5} & 0 & 0 & 0 & 0 & 9.29 \times 10^{4} & -1.393 \times 10^{5} \\ 0 & 0 & 2.295 \times 10^{4} & 6.885 \times 10^{4} & 0 & 0 & -2.295 \times 10^{4} & -6.885 \times 10^{4} \\ 0 & 0 & 6.885 \times 10^{4} & 2.065 \times 10^{5} & 0 & 0 & -6.885 \times 10^{4} & -2.065 \times 10^{5} \\ 0 & 0 & 0 & 0 & 8.553 \times 10^{4} & 8.553 \times 10^{4} & -8.553 \times 10^{4} & -8.553 \times 10^{4} \\ 0 & 0 & 0 & 0 & 8.553 \times 10^{4} & 8.553 \times 10^{4} & -8.553 \times 10^{4} & -8.553 \times 10^{4} \\ -6.193 \times 10^{4} & 9.29 \times 10^{4} & -2.295 \times 10^{4} & -6.885 \times 10^{4} & -8.553 \times 10^{4} & -8.553 \times 10^{4} & 1.704 \times 10^{5} & 6.148 \times 10^{4} \\ 9.29 \times 10^{4} & -1.393 \times 10^{5} & -6.885 \times 10^{4} & -2.065 \times 10^{5} & -8.553 \times 10^{4} & -8.553 \times 10^{4} & 6.148 \times 10^{4} & 4.314 \times 10^{5} \end{pmatrix}$$

$$u = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ p7 \\ p8 \end{pmatrix} \qquad U = \begin{pmatrix} R_{1v} \\ R_{1l} \\ R_{2v} \\ R_{2l} \\ R_{3v} \\ R_{3l} \\ 10 \\ 15 \end{pmatrix}$$

K er OK med FElt.

$K_{mod} := \begin{pmatrix} K_{7,7} & K_{7,8} \\ K_{8,7} & K_{8,8} \end{pmatrix}$

$u_{mod} = \begin{pmatrix} p7 \\ p8 \end{pmatrix}$ $U_{mod} := \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

$u_{mod} := K_{mod}^{-1} \cdot U_{mod}$ $u_{mod} = \begin{pmatrix} 4.864 \times 10^{-5} \\ 2.784 \times 10^{-5} \end{pmatrix}$ $u_{7,1} := {u_{mod}}_{1,1}$ $u_{8,1} := {u_{mod}}_{2,1}$

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$$K\cdot u = \begin{pmatrix} -0.426 \\ 0.639 \\ -3.033 \\ -9.098 \\ -6.541 \\ -6.541 \\ 10 \\ 15 \end{pmatrix} \qquad u = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4.864\times 10^{-5} \\ 2.784\times 10^{-5} \end{pmatrix}$$

$$K1 = \begin{pmatrix} 6.193\times 10^{4} & -9.29\times 10^{4} & -6.193\times 10^{4} & 9.29\times 10^{4} \\ -9.29\times 10^{4} & 1.393\times 10^{5} & 9.29\times 10^{4} & -1.393\times 10^{5} \\ -6.193\times 10^{4} & 9.29\times 10^{4} & 6.193\times 10^{4} & -9.29\times 10^{4} \\ 9.29\times 10^{4} & -1.393\times 10^{5} & -9.29\times 10^{4} & 1.393\times 10^{5} \end{pmatrix} \qquad u1 := \begin{pmatrix} 0 \\ 0 \\ u_{7,1} \\ u_{8,1} \end{pmatrix}$$

$$K1\cdot u1 = \begin{pmatrix} -0.426 \\ 0.639 \\ 0.426 \\ -0.639 \end{pmatrix} \qquad \sqrt{0.426^2 + 0.639^2} = 0.768$$

$$K2 = \begin{pmatrix} 2.295\times 10^{4} & 6.885\times 10^{4} & -2.295\times 10^{4} & -6.885\times 10^{4} \\ 6.885\times 10^{4} & 2.065\times 10^{5} & -6.885\times 10^{4} & -2.065\times 10^{5} \\ -2.295\times 10^{4} & -6.885\times 10^{4} & 2.295\times 10^{4} & 6.885\times 10^{4} \\ -6.885\times 10^{4} & -2.065\times 10^{5} & 6.885\times 10^{4} & 2.065\times 10^{5} \end{pmatrix} \qquad u2 := \begin{pmatrix} 0 \\ 0 \\ u_{7,1} \\ u_{8,1} \end{pmatrix}$$

$$K2\cdot u2 = \begin{pmatrix} -3.033 \\ -9.098 \\ 3.033 \\ 9.098 \end{pmatrix} \qquad \sqrt{3.033^2 + 9.098^2} = 9.59$$

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$$K3 = \begin{pmatrix} 8.553\times10^{4} & 8.553\times10^{4} & -8.553\times10^{4} & -8.553\times10^{4} \\ 8.553\times10^{4} & 8.553\times10^{4} & -8.553\times10^{4} & -8.553\times10^{4} \\ -8.553\times10^{4} & -8.553\times10^{4} & 8.553\times10^{4} & 8.553\times10^{4} \\ -8.553\times10^{4} & -8.553\times10^{4} & 8.553\times10^{4} & 8.553\times10^{4} \end{pmatrix}$$

$$u3 := \begin{pmatrix} 0 \\ 0 \\ u_{7,1} \\ u_{8,1} \end{pmatrix}$$

$$K3\cdot u3 = \begin{pmatrix} -6.541 \\ -6.541 \\ 6.541 \\ 6.541 \end{pmatrix}$$

$$\sqrt{6.541^{2}\cdot 2} = 9.25$$