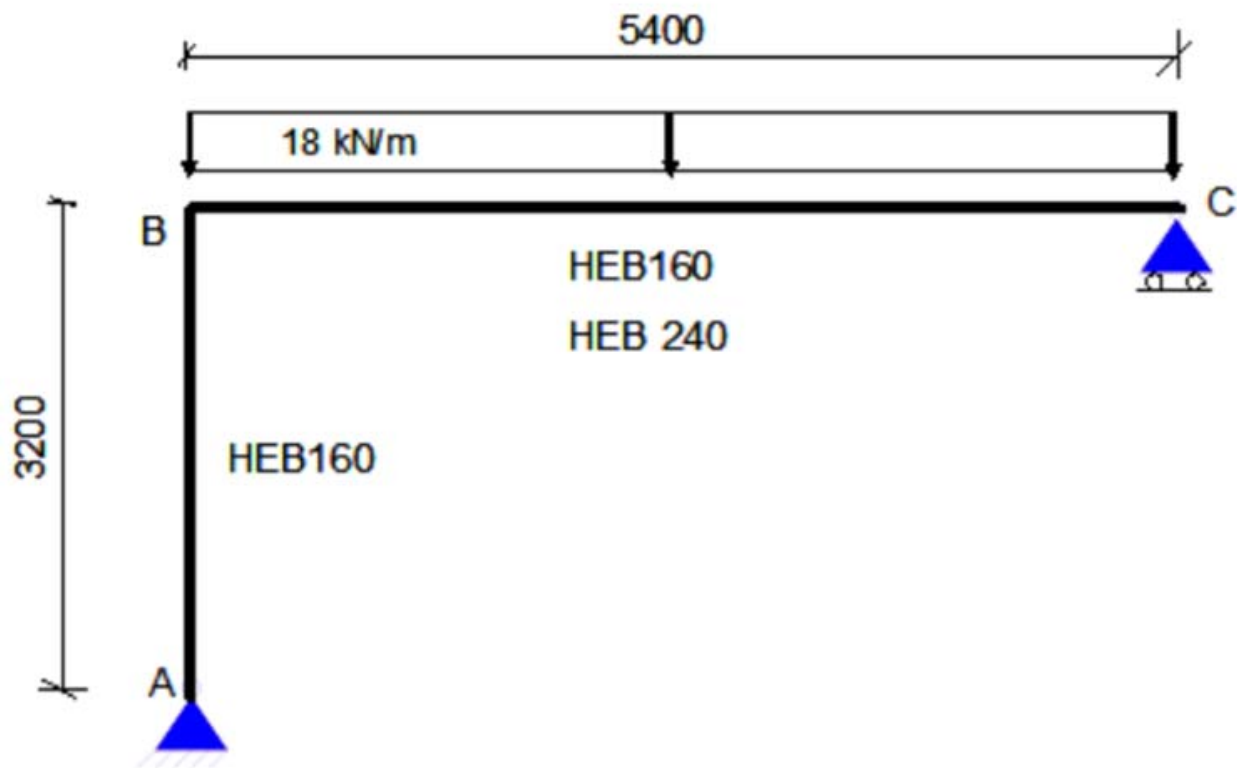


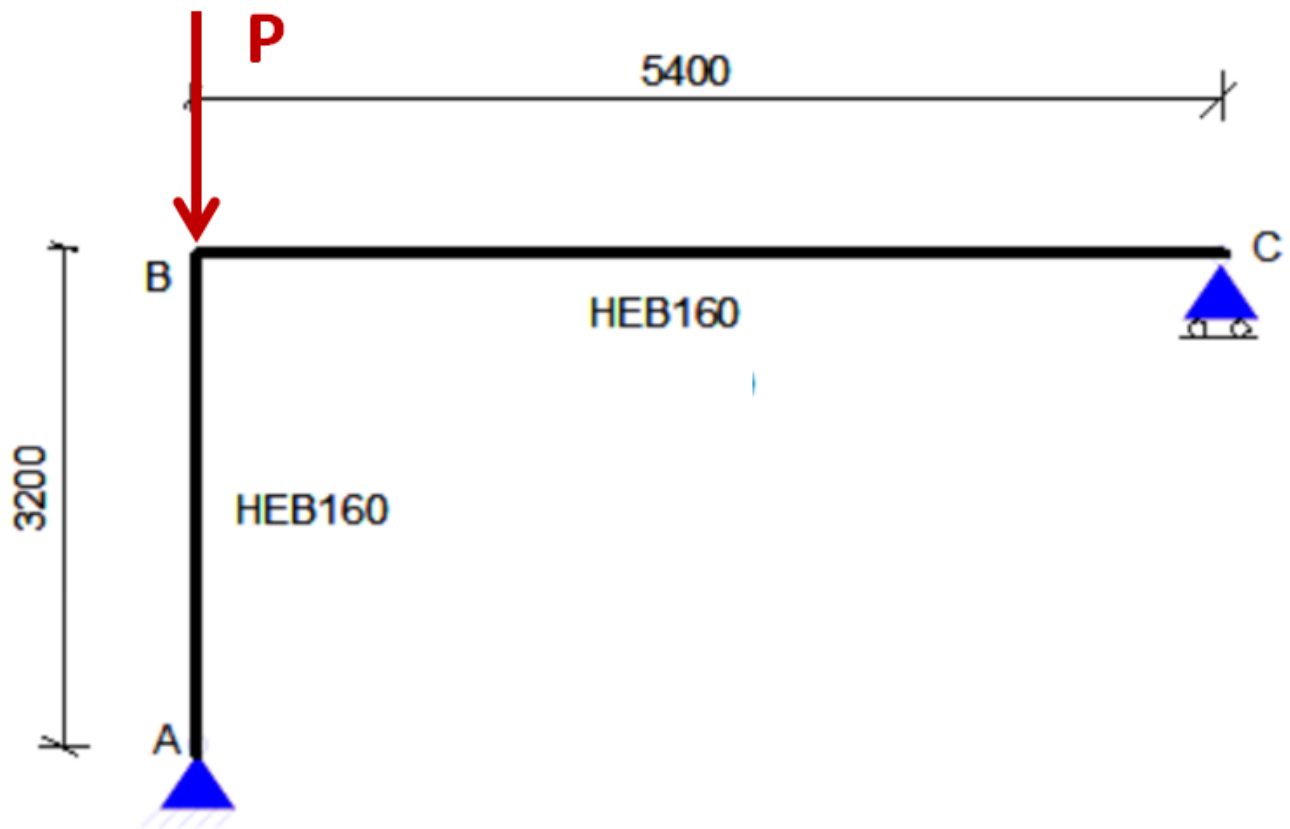
### Opgave 4.32.

#### Løsningsforslag til kritisk last og søjlelængde.

Som en lidt grov forenkling underopdeles elementerne ikke.  
Derfor vil der være usikkerhed på resultatet.



Ved beregning af den kritiske last anvendes dette system:



[> restart  
 Loading [LinearAlgebra](#)  
 UseSystem(system, opts)

with(LinearAlgebra) :

**Forudsætninger**

$$\begin{aligned} \varphi1 &:= 2 \cdot \left(\frac{x}{L}\right)^3 - 3 \cdot \left(\frac{x}{L}\right)^2 + 1 : \\ \varphi2 &:= -x \cdot \left( \left(\frac{x}{L}\right)^2 - 2 \cdot \left(\frac{x}{L}\right) + 1 \right) : \\ \varphi3 &:= -2 \cdot \left(\frac{x}{L}\right)^3 + 3 \cdot \left(\frac{x}{L}\right)^2 : \\ \varphi4 &:= -x \cdot \left( \left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right) \right) : \end{aligned}$$

**Lastgeometrimatricen for et rammeelement:**

$$K_{\sigma} = \frac{N}{30 \cdot L} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & -3 \cdot L & 0 & -36 & -3 \cdot L \\ 0 & -3 \cdot L & 4 \cdot L^2 & 0 & 3 \cdot L & -L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & 3 \cdot L & 0 & 36 & 3 \cdot L \\ 0 & -3 \cdot L & -L^2 & 0 & 3 \cdot L & 4 \cdot L^2 \end{bmatrix} :$$

**Stivhedsmatrice for et ramme element:**

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12IE}{L^3} & -\frac{6IE}{L^2} & 0 & -\frac{12IE}{L^3} & -\frac{6IE}{L^2} \\ 0 & -\frac{6IE}{L^2} & \frac{4IE}{L} & 0 & \frac{6IE}{L^2} & \frac{2IE}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12IE}{L^3} & \frac{6IE}{L^2} & 0 & \frac{12IE}{L^3} & \frac{6IE}{L^2} \\ 0 & -\frac{6IE}{L^2} & \frac{2IE}{L} & 0 & \frac{6IE}{L^2} & \frac{4IE}{L} \end{bmatrix} :$$

**Transformationsmatrice.**

$$T = \begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\nu) & \sin(\nu) & 0 \\ 0 & 0 & 0 & -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

▼ **Stivhedsmatricer lokalt**

$$\begin{aligned}
K_{II} &:= \begin{bmatrix} \frac{EA I}{L I} & 0 & 0 & -\frac{EA I}{L I} & 0 & 0 \\ 0 & \frac{12 E I I}{L I^3} & -\frac{6 E I I}{L I^2} & 0 & -\frac{12 E I I}{L I^3} & -\frac{6 E I I}{L I^2} \\ 0 & -\frac{6 E I I}{L I^2} & \frac{4 E I I}{L I} & 0 & \frac{6 E I I}{L I^2} & \frac{2 E I I}{L I} \\ -\frac{EA I}{L I} & 0 & 0 & \frac{EA I}{L I} & 0 & 0 \\ 0 & -\frac{12 E I I}{L I^3} & \frac{6 E I I}{L I^2} & 0 & \frac{12 E I I}{L I^3} & \frac{6 E I I}{L I^2} \\ 0 & -\frac{6 E I I}{L I^2} & \frac{2 E I I}{L I} & 0 & \frac{6 E I I}{L I^2} & \frac{4 E I I}{L I} \end{bmatrix} \\
& \begin{bmatrix} \frac{EA I}{L I} & 0 & 0 & -\frac{EA I}{L I} & 0 & 0 \\ 0 & \frac{12 E I I}{L I^3} & -\frac{6 E I I}{L I^2} & 0 & -\frac{12 E I I}{L I^3} & -\frac{6 E I I}{L I^2} \\ 0 & -\frac{6 E I I}{L I^2} & \frac{4 E I I}{L I} & 0 & \frac{6 E I I}{L I^2} & \frac{2 E I I}{L I} \\ -\frac{EA I}{L I} & 0 & 0 & \frac{EA I}{L I} & 0 & 0 \\ 0 & -\frac{12 E I I}{L I^3} & \frac{6 E I I}{L I^2} & 0 & \frac{12 E I I}{L I^3} & \frac{6 E I I}{L I^2} \\ 0 & -\frac{6 E I I}{L I^2} & \frac{2 E I I}{L I} & 0 & \frac{6 E I I}{L I^2} & \frac{4 E I I}{L I} \end{bmatrix} \\
K_{II \sigma} &:= \frac{N}{30 \cdot L I} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & -3 \cdot L I & 0 & -36 & -3 \cdot L I \\ 0 & -3 \cdot L I & 4 \cdot L I^2 & 0 & 3 \cdot L I & -L I^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & 3 \cdot L I & 0 & 36 & 3 \cdot L I \\ 0 & -3 \cdot L I & -L I^2 & 0 & 3 \cdot L I & 4 \cdot L I^2 \end{bmatrix}
\end{aligned} \tag{2.1}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{5} \frac{N}{LI} & -\frac{1}{10} N & 0 & -\frac{6}{5} \frac{N}{LI} & -\frac{1}{10} N \\ 0 & -\frac{1}{10} N & \frac{2}{15} NLI & 0 & \frac{1}{10} N & -\frac{1}{30} NLI \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{6}{5} \frac{N}{LI} & \frac{1}{10} N & 0 & \frac{6}{5} \frac{N}{LI} & \frac{1}{10} N \\ 0 & -\frac{1}{10} N & -\frac{1}{30} NLI & 0 & \frac{1}{10} N & \frac{2}{15} NLI \end{bmatrix}$$

(2.2)

$$K_{21} := \begin{bmatrix} \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 \\ 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} \\ -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 \\ 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 \\ 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} \\ -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 \\ 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} \\ 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2} \end{bmatrix}$$

(2.3)

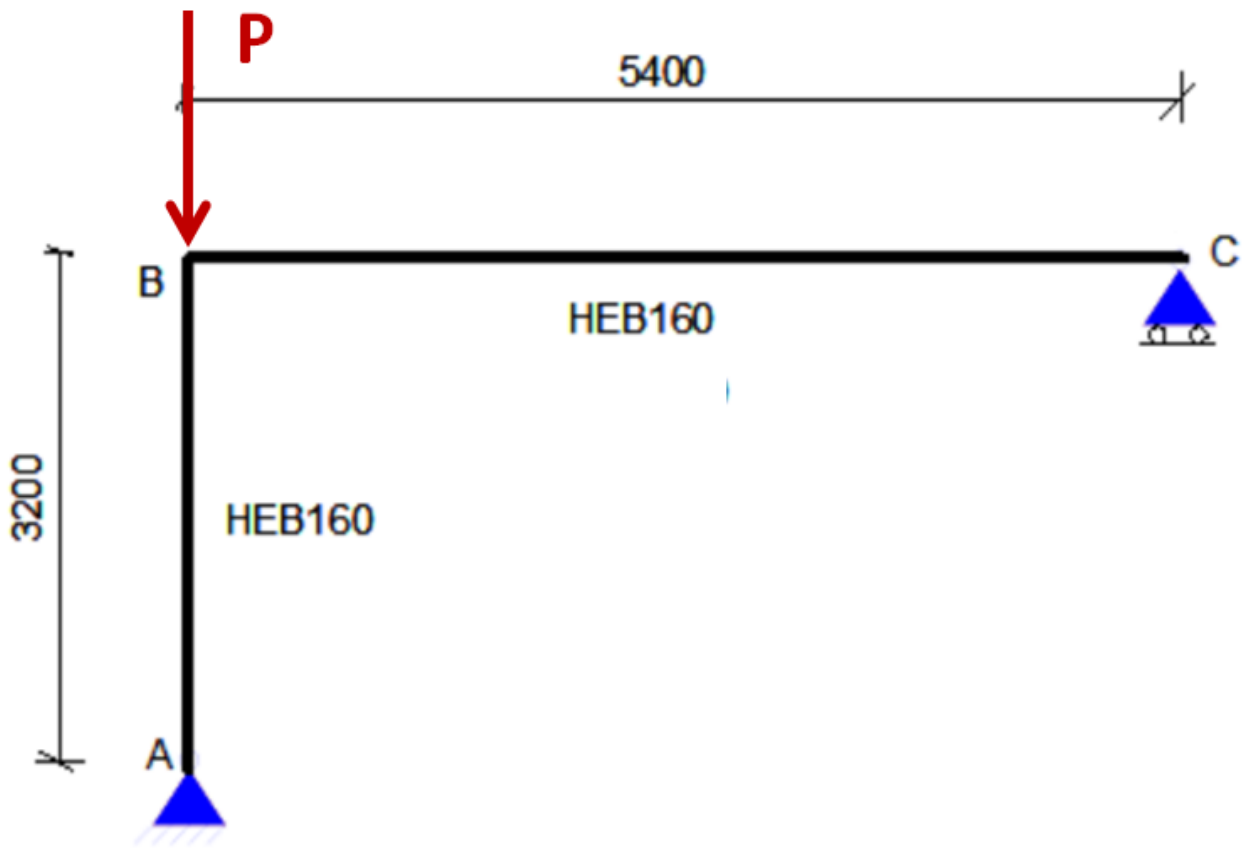
$$K_{21}_{\sigma} := \frac{N}{30 \cdot L^2} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & -3 \cdot L^2 & 0 & -36 & -3 \cdot L^2 \\ 0 & -3 \cdot L^2 & 4 \cdot L^2^2 & 0 & 3 \cdot L^2 & -L^2^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & 3 \cdot L^2 & 0 & 36 & 3 \cdot L^2 \\ 0 & -3 \cdot L^2 & -L^2^2 & 0 & 3 \cdot L^2 & 4 \cdot L^2^2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{5} \frac{N}{L^2} & -\frac{1}{10} N & 0 & -\frac{6}{5} \frac{N}{L^2} & -\frac{1}{10} N \\ 0 & -\frac{1}{10} N & \frac{2}{15} NL^2 & 0 & \frac{1}{10} N & -\frac{1}{30} NL^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{6}{5} \frac{N}{L^2} & \frac{1}{10} N & 0 & \frac{6}{5} \frac{N}{L^2} & \frac{1}{10} N \\ 0 & -\frac{1}{10} N & -\frac{1}{30} NL^2 & 0 & \frac{1}{10} N & \frac{2}{15} NL^2 \end{bmatrix} \quad (2.4)$$

### Stivhedsmatricer globalt

$$T := \begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\nu) & \sin(\nu) & 0 \\ 0 & 0 & 0 & -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\nu) & \sin(\nu) & 0 \\ 0 & 0 & 0 & -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.1)$$



$$vI := \frac{\pi}{2}$$

$$\frac{1}{2} \pi$$

(3.2)

$$T1 := \begin{bmatrix} \cos(vI) & \sin(vI) & 0 & 0 & 0 & 0 \\ -\sin(vI) & \cos(vI) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(vI) & \sin(vI) & 0 \\ 0 & 0 & 0 & -\sin(vI) & \cos(vI) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3.3)

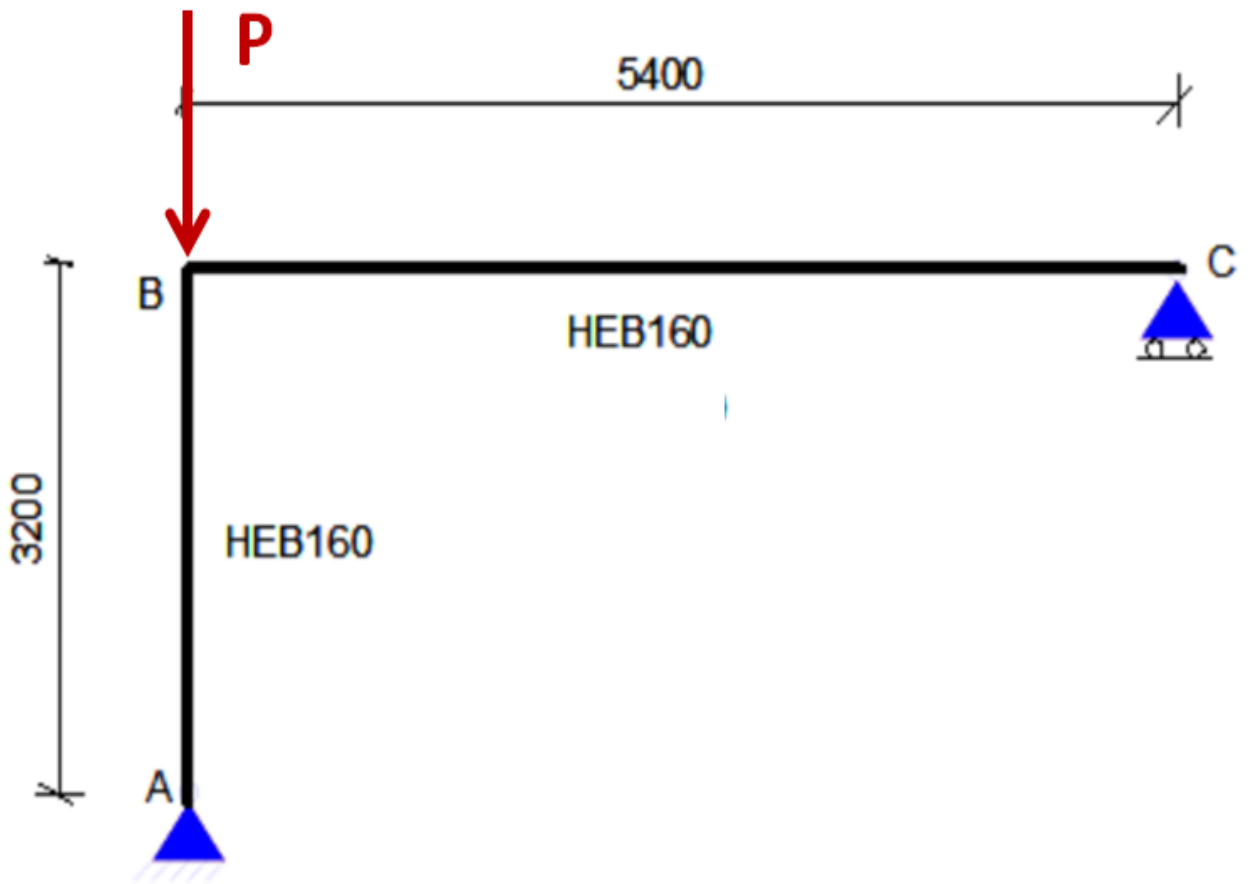
$$K12 := T1^+ . K11 . T1$$

$$\begin{bmatrix}
 \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\
 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\
 -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} \\
 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L}
 \end{bmatrix} \quad (3.4)$$

$$K12_{\sigma} := T1^+ \cdot K11_{\sigma} \cdot T1$$

$$\begin{bmatrix}
 \frac{6}{5} \frac{N}{L} & 0 & \frac{1}{10} N & -\frac{6}{5} \frac{N}{L} & 0 & \frac{1}{10} N \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & \frac{2}{15} NLI & -\frac{1}{10} N & 0 & -\frac{1}{30} NLI \\
 -\frac{6}{5} \frac{N}{L} & 0 & -\frac{1}{10} N & \frac{6}{5} \frac{N}{L} & 0 & -\frac{1}{10} N \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & -\frac{1}{30} NLI & -\frac{1}{10} N & 0 & \frac{2}{15} NLI
 \end{bmatrix} \quad (3.5)$$





$$v2 := 0$$

$$0$$

(3.6)

$$T2 := \begin{bmatrix} \cos(v2) & \sin(v2) & 0 & 0 & 0 & 0 \\ -\sin(v2) & \cos(v2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(v2) & \sin(v2) & 0 \\ 0 & 0 & 0 & -\sin(v2) & \cos(v2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3.7)

$$K22 := T2^+ . K21 . T2$$

$$\begin{bmatrix}
 \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 \\
 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} \\
 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} \\
 -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 \\
 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} \\
 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2}
 \end{bmatrix}$$

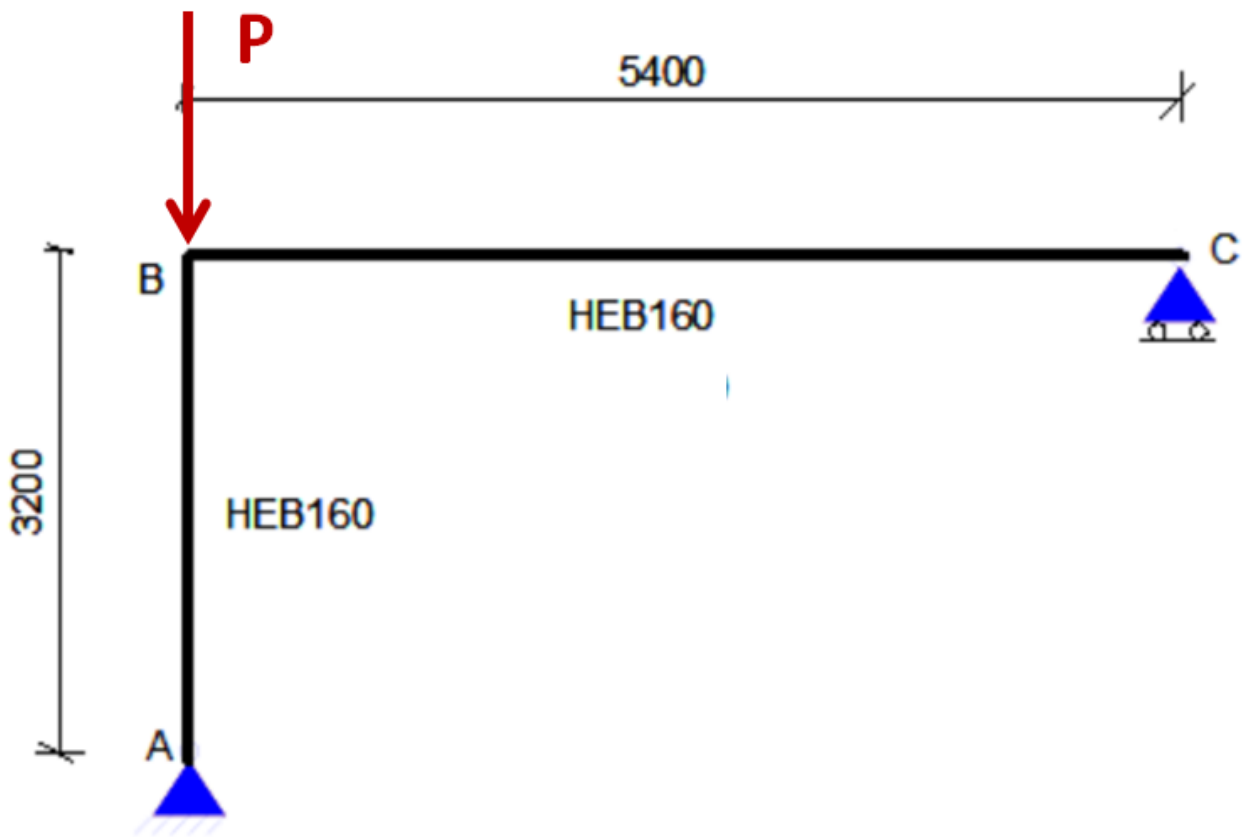
(3.8)

$$K22_{\sigma} := T1^+ \cdot K21_{\sigma} \cdot T1$$

$$\begin{bmatrix}
 \frac{6}{5} \frac{N}{L2} & 0 & \frac{1}{10} N & -\frac{6}{5} \frac{N}{L2} & 0 & \frac{1}{10} N \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & \frac{2}{15} NL2 & -\frac{1}{10} N & 0 & -\frac{1}{30} NL2 \\
 -\frac{6}{5} \frac{N}{L2} & 0 & -\frac{1}{10} N & \frac{6}{5} \frac{N}{L2} & 0 & -\frac{1}{10} N \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & -\frac{1}{30} NL2 & -\frac{1}{10} N & 0 & \frac{2}{15} NL2
 \end{bmatrix}$$

(3.9)

▼ Det samlede system



$$\begin{aligned}
 & \left[ \begin{array}{c} & P_1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; ; P_2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} : \\
 & \left[ \right]
 \end{aligned}$$

$$KI := P_1 \cdot KI2 \cdot P_1^+$$

$$\begin{bmatrix}
 \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & 0 & 0 & 0 \\
 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & 0 & 0 & 0 \\
 -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & 0 & 0 & 0 \\
 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 \\
 \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}$$

(4.1)

$$K1_{\sigma} := P_1 \cdot K12_{\sigma} \cdot P_1^+$$

$$\begin{bmatrix}
 \frac{6}{5} \frac{N}{L} & 0 & \frac{1}{10} N & -\frac{6}{5} \frac{N}{L} & 0 & \frac{1}{10} N & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & \frac{2}{15} NLI & -\frac{1}{10} N & 0 & -\frac{1}{30} NLI & 0 & 0 & 0 \\
 -\frac{6}{5} \frac{N}{L} & 0 & -\frac{1}{10} N & \frac{6}{5} \frac{N}{L} & 0 & -\frac{1}{10} N & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{10} N & 0 & -\frac{1}{30} NLI & -\frac{1}{10} N & 0 & \frac{2}{15} NLI & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}$$

(4.2)

$$K2 := P_2 \cdot K22 \cdot P_2^+$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{EA2}{L2} & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} \\
0 & 0 & 0 & 0 & -\frac{6EI2}{L2^2} & \frac{4EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{2EI2}{L2} \\
0 & 0 & 0 & -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} & 0 & \frac{12EI2}{L2^3} & \frac{6EI2}{L2^2} \\
0 & 0 & 0 & 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{6EI2}{L2^2} & \frac{4EI2}{L2}
\end{bmatrix}$$

(4.3)

$$K2_{\sigma} := P_2 \cdot K22_{\sigma} \cdot P_2^+$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{6}{5} \frac{N}{L2} & 0 & \frac{1}{10} N & -\frac{6}{5} \frac{N}{L2} & 0 & \frac{1}{10} N \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{10} N & 0 & \frac{2}{15} NL2 & -\frac{1}{10} N & 0 & -\frac{1}{30} NL2 \\
0 & 0 & 0 & -\frac{6}{5} \frac{N}{L2} & 0 & -\frac{1}{10} N & \frac{6}{5} \frac{N}{L2} & 0 & -\frac{1}{10} N \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{10} N & 0 & -\frac{1}{30} NL2 & -\frac{1}{10} N & 0 & \frac{2}{15} NL2
\end{bmatrix}$$

(4.4)

$$K := K1 + K2$$

$$\begin{bmatrix}
\left[ \left[ \frac{12EI1}{L1^3}, 0, \frac{6EI1}{L1^2}, -\frac{12EI1}{L1^3}, 0, \frac{6EI1}{L1^2}, 0, 0, 0 \right], \right. \\
\left[ 0, \frac{EA1}{L1}, 0, 0, -\frac{EA1}{L1}, 0, 0, 0, 0 \right], \\
\left. \left[ \frac{6EI1}{L1^2}, 0, \frac{4EI1}{L1}, -\frac{6EI1}{L1^2}, 0, \frac{2EI1}{L1}, 0, 0, 0 \right] \right]$$

(4.5)

$$\begin{aligned}
& \left[ -\frac{12EI}{L^3}, 0, -\frac{6EI}{L^2}, \frac{12EI}{L^3} + \frac{EA2}{L2}, 0, -\frac{6EI}{L^2}, -\frac{EA2}{L2}, 0, 0 \right], \\
& \left[ 0, -\frac{EA1}{L}, 0, 0, \frac{EA1}{L} + \frac{12EI2}{L^3}, -\frac{6EI2}{L^2}, 0, -\frac{12EI2}{L^3}, -\frac{6EI2}{L^2} \right], \\
& \left[ \frac{6EI}{L^2}, 0, \frac{2EI}{L}, -\frac{6EI}{L^2}, -\frac{6EI2}{L^2}, \frac{4EI}{L} + \frac{4EI2}{L2}, 0, \frac{6EI2}{L^2}, \frac{2EI2}{L2} \right], \\
& \left[ 0, 0, 0, -\frac{EA2}{L2}, 0, 0, \frac{EA2}{L2}, 0, 0 \right], \\
& \left[ 0, 0, 0, 0, -\frac{12EI2}{L^3}, \frac{6EI2}{L^2}, 0, \frac{12EI2}{L^3}, \frac{6EI2}{L^2} \right], \\
& \left[ 0, 0, 0, 0, -\frac{6EI2}{L^2}, \frac{2EI2}{L2}, 0, \frac{6EI2}{L^2}, \frac{4EI2}{L2} \right]
\end{aligned}$$

$$K_{\sigma} := KI_{\sigma} + K2_{\sigma}$$

$$\left[ \left[ \frac{6}{5} \frac{N}{L}, 0, \frac{1}{10} N, -\frac{6}{5} \frac{N}{L}, 0, \frac{1}{10} N, 0, 0, 0 \right], \right. \tag{4.6}$$

$$\left[ 0, 0, 0, 0, 0, 0, 0, 0 \right],$$

$$\left[ \frac{1}{10} N, 0, \frac{2}{15} NLI, -\frac{1}{10} N, 0, -\frac{1}{30} NLI, 0, 0, 0 \right],$$

$$\left[ -\frac{6}{5} \frac{N}{L}, 0, -\frac{1}{10} N, \frac{6}{5} \frac{N}{L} + \frac{6}{5} \frac{N}{L2}, 0, 0, -\frac{6}{5} \frac{N}{L2}, 0, \frac{1}{10} N \right],$$

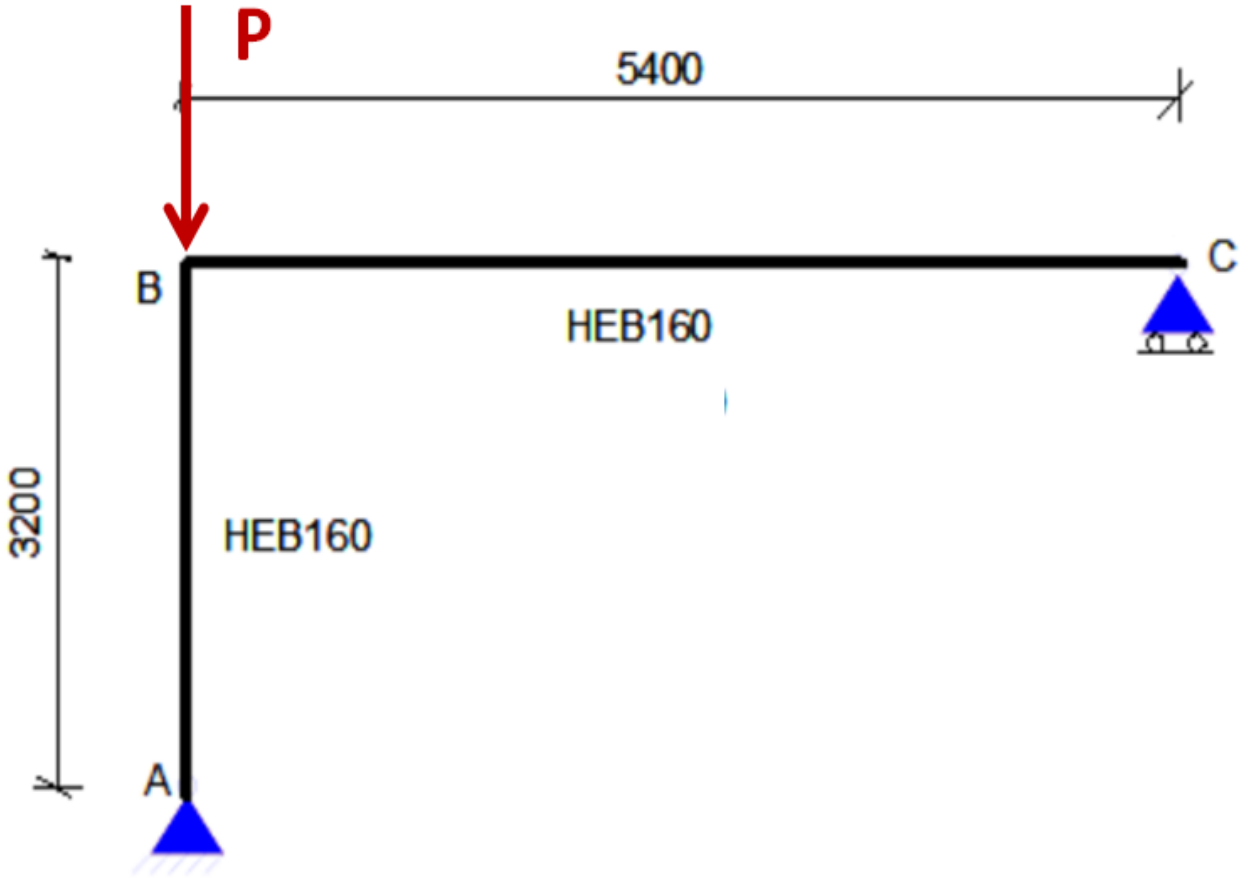
$$\left[ 0, 0, 0, 0, 0, 0, 0, 0 \right],$$

$$\left[ \frac{1}{10} N, 0, -\frac{1}{30} NLI, 0, 0, \frac{2}{15} NLI + \frac{2}{15} NL2, -\frac{1}{10} N, 0, -\frac{1}{30} NL2 \right],$$

$$\left[ 0, 0, 0, -\frac{6}{5} \frac{N}{L2}, 0, -\frac{1}{10} N, \frac{6}{5} \frac{N}{L2}, 0, -\frac{1}{10} N \right],$$

$$\left[ 0, 0, 0, 0, 0, 0, 0, 0 \right],$$

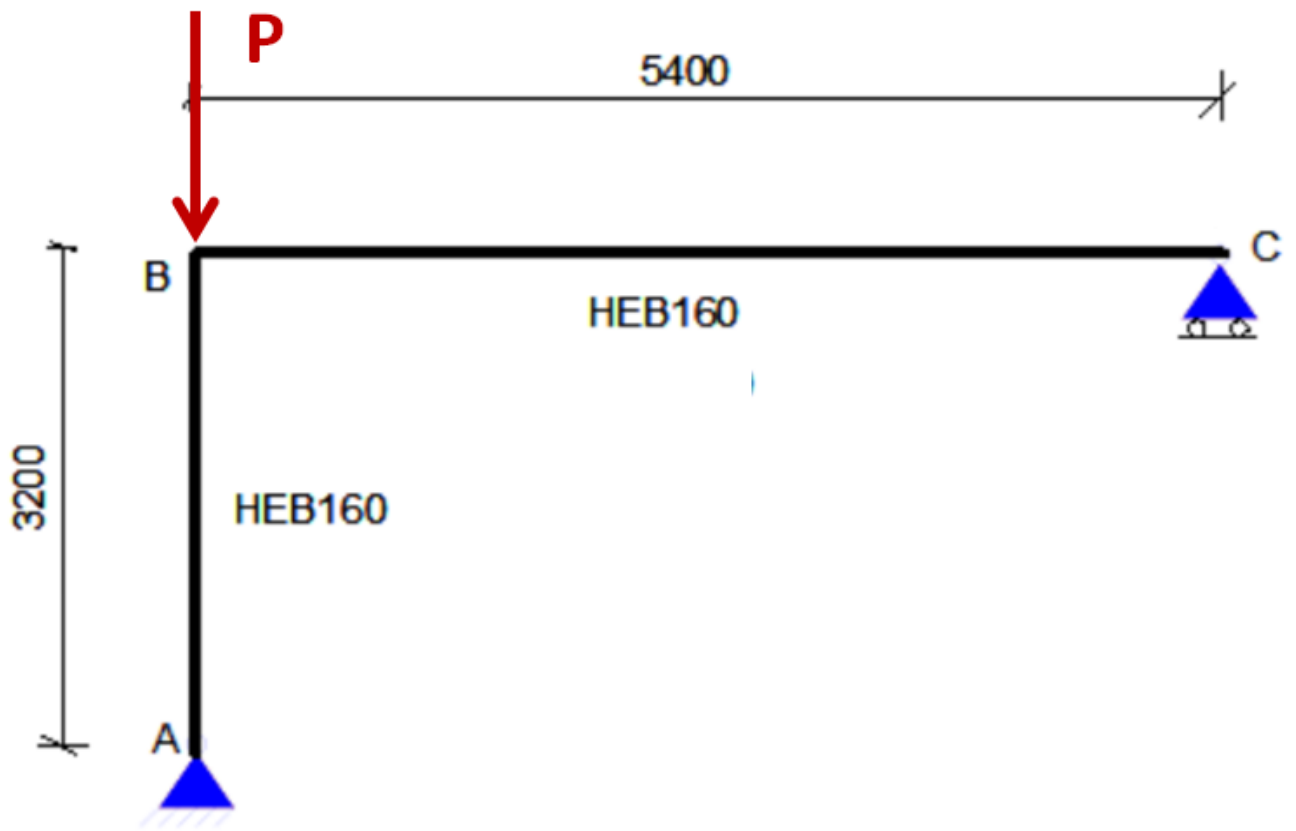
$$\left[ 0, 0, 0, \frac{1}{10} N, 0, -\frac{1}{30} NL2, -\frac{1}{10} N, 0, \frac{2}{15} NL2 \right]$$



$$u := \begin{bmatrix} 0 \\ 0 \\ rA \\ uBv \\ uBl \\ rB \\ uCv \\ 0 \\ rC \end{bmatrix} :$$

$$U := \begin{bmatrix} Av \\ Al \\ 0 \\ 0 \\ P \\ 0 \\ 0 \\ Lc \\ 0 \end{bmatrix} :$$

▼ Modificeret system





$$pm := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} :$$

$$K_{mod} := pm^+ \cdot K \cdot pm$$

$$\begin{bmatrix} \frac{4EI1}{L1} & -\frac{6EI1}{L1^2} & 0 & \frac{2EI1}{L1} & 0 & 0 \\ -\frac{6EI1}{L1^2} & \frac{12EI1}{L1^3} + \frac{EA2}{L2} & 0 & -\frac{6EI1}{L1^2} & -\frac{EA2}{L2} & 0 \\ 0 & 0 & \frac{EA1}{L1} + \frac{12EI2}{L2^3} & -\frac{6EI2}{L2^2} & 0 & -\frac{6EI2}{L2^2} \\ \frac{2EI1}{L1} & -\frac{6EI1}{L1^2} & -\frac{6EI2}{L2^2} & \frac{4EI1}{L1} + \frac{4EI2}{L2} & 0 & \frac{2EI2}{L2} \\ 0 & -\frac{EA2}{L2} & 0 & 0 & \frac{EA2}{L2} & 0 \\ 0 & 0 & -\frac{6EI2}{L2^2} & \frac{2EI2}{L2} & 0 & \frac{4EI2}{L2} \end{bmatrix} \quad (1)$$

$$K_{\sigma mod} := pm^+ \cdot K_{\sigma} \cdot pm$$

$$\begin{bmatrix} \frac{2}{15} NLI & -\frac{1}{10} N & 0 & -\frac{1}{30} NLI & 0 & 0 \\ -\frac{1}{10} N & \frac{6}{5} \frac{N}{L1} + \frac{6}{5} \frac{N}{L2} & 0 & 0 & -\frac{6}{5} \frac{N}{L2} & \frac{1}{10} N \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{30} NLI & 0 & 0 & \frac{2}{15} NLI + \frac{2}{15} NL2 & -\frac{1}{10} N & -\frac{1}{30} NL2 \\ 0 & -\frac{6}{5} \frac{N}{L2} & 0 & -\frac{1}{10} N & \frac{6}{5} \frac{N}{L2} & -\frac{1}{10} N \\ 0 & \frac{1}{10} N & 0 & -\frac{1}{30} NL2 & -\frac{1}{10} N & \frac{2}{15} NL2 \end{bmatrix} \quad (2)$$

## Eksempel med HEB160

$$E := 210000 \quad 210000 \quad (6.1)$$

$$A1 := 5425 \quad 5425 \quad (6.2)$$

$$A2 := A1 \quad 5425 \quad (6.3)$$

$$I1 := 24.9 \cdot 10^6 \quad 2.49000000 \cdot 10^7 \quad (6.4)$$

$$I2 := I1 \quad 2.49000000 \cdot 10^7 \quad (6.5)$$

$$L1 := 3200 \quad 3200 \quad (6.6)$$

$$L2 := 5400 \quad 5400 \quad (6.7)$$

$$P := 100000 \quad 100000 \quad (6.8)$$

$$N := P \quad 100000 \quad (6.9)$$

$$Arb := Kmod - \lambda \cdot K\sigma mod$$

$$\left[ \left[ -\frac{128000000}{3} \lambda + 6.536250000 \cdot 10^9, 10000 \lambda - 3.063867187 \cdot 10^6, 0, \frac{320000000}{3} \lambda + 3.268125000 \cdot 10^9, 0, 0 \right], \right. \quad (6.10)$$

$$\left[ 10000 \lambda - 3.063867187 \cdot 10^6, -\frac{1075}{18} \lambda + 2.128871392 \cdot 10^5, 0, -3.063867187 \cdot 10^6, \frac{200}{9} \lambda - \frac{1898750}{9}, -10000 \lambda \right],$$

$$\left[ 0, 0, 3.564141161 \cdot 10^5, -1.075925926 \cdot 10^6, 0, -1.075925926 \cdot 10^6 \right],$$

$$\left[ \frac{320000000}{3} \lambda + 3.268125000 \cdot 10^9, -3.063867187 \cdot 10^6, -1.075925926 \cdot 10^6, -\frac{344000000}{3} \lambda + 1.040958333 \cdot 10^{10}, 10000 \lambda, 18000000 \lambda + 1.936666667 \cdot 10^9 \right],$$

$$\left[ 0, \frac{200}{9} \lambda - \frac{1898750}{9}, 0, 10000 \lambda, -\frac{200}{9} \lambda + \frac{1898750}{9}, 10000 \lambda \right],$$

$$\left[ 0, -10000 \lambda, -1.075925926 \cdot 10^6, 18000000 \lambda + 1.936666667 \cdot 10^9, 10000 \lambda, -72000000 \lambda \right]$$

$$+ 3.873333333 \cdot 10^9]]$$

$Cra := \text{LinearAlgebra}[\text{Determinant}](\text{Arb})$

$$2.64450250449715 \cdot 10^{42} + 6.86953559858171 \cdot 10^{31} \lambda^5 - 2.92080924888910 \cdot 10^{38} \lambda^3 \quad (6.11)$$

$$+ 8.65163095685100 \cdot 10^{35} \lambda^4 + 2.61248173381365 \cdot 10^{40} \lambda^2 - 6.63578275047232 \cdot 10^{41} \lambda$$

$Cra = 0$

$$2.64450250449715 \cdot 10^{42} + 6.86953559858171 \cdot 10^{31} \lambda^5 - 2.92080924888910 \cdot 10^{38} \lambda^3 \quad (6.12)$$

$$+ 8.65163095685100 \cdot 10^{35} \lambda^4 + 2.61248173381365 \cdot 10^{40} \lambda^2 - 6.63578275047232 \cdot 10^{41} \lambda$$

$$= 0$$

$\text{solve}(\{Cra = 0\}, [\lambda])$

$$[[\lambda = 4.868200633], [\lambda = 34.15855518], [\lambda = 87.48863753], [\lambda = 204.7160624], [\lambda = -12925.43209]] \quad (6.13)$$

$Pcr := 4.87 \cdot P$

$$4.8700000 \cdot 10^5 \quad (6.14)$$

$$Ls := \pi \cdot \sqrt{\frac{E \cdot I I}{Pcr}}$$

$$10294.25005 \quad (6.15)$$